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AUTHOR Renaud, Albert J.
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ABSTRACT

This guidebook reports the results of a three-year study involving the use of paraprofessionals with mathematics students in the La Mesa-Spring Valley School District (California), and outlines a program for initiating a mathematics paraprofessional program. Over 1200 students in grades 4-7 participated in this project to determine the effects of paraprofessional assistance upon student achievement and attitudes in mathematics. The study included pre- and posttesting of both attitude and achievement for all students involved in the project. After the paraprofessionals were hired, they were randomly assigned to the trained or untrained category, then randomly assigned to a selected school and grade level. The three major project components and four objectives of the project are listed. Also included in this guide are: (1) guidelines for selecting paraprofessionals; (2) guidelines for the principal and teacher when working with paraprofessionals; (3) student statements regarding the involvement of the mathematics paraprofessionals; (4) a definition of a mathematics paraprofessional; (5) a general outline and materials for preservice training workshops covering numbers and operations, geometry, measurement, logic, functions, probability and statistics, and problem solving; (6) an evaluation of the project; and (7) the analysis procedures and obtained results, which are discussed in relation to the corresponding evaluation instrument. Appendices include four of the five evaluation instruments used and a list of mathematics materials applicable for mathematics workshops. (JW)

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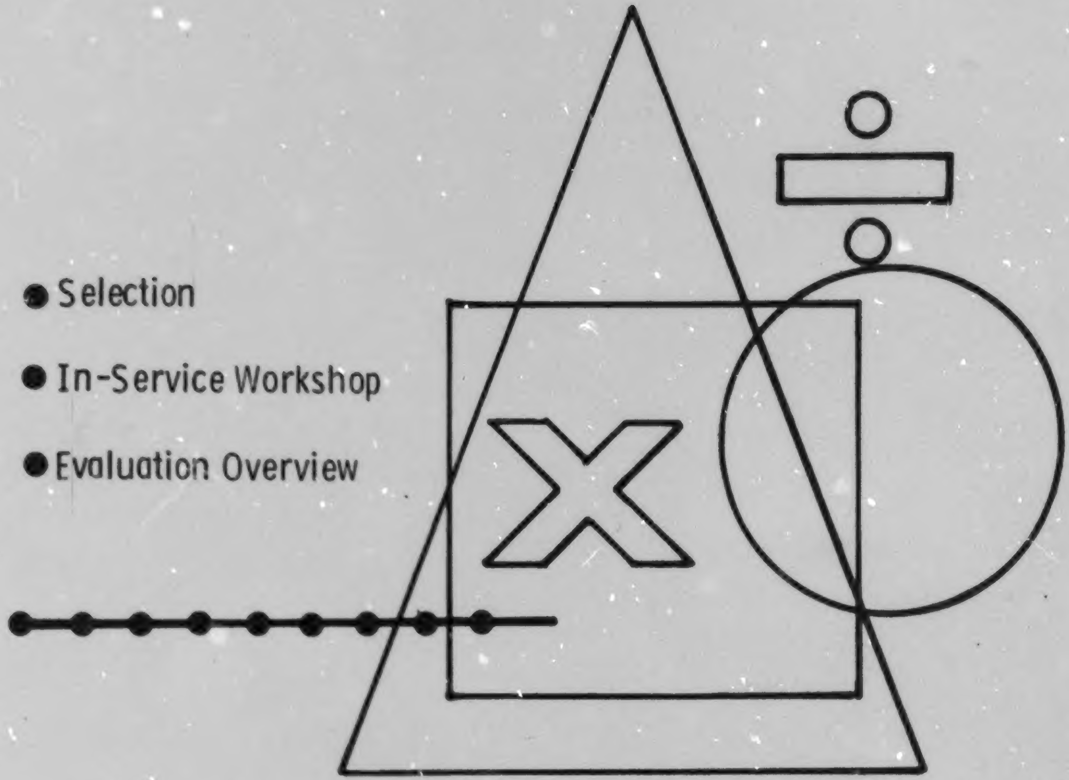

PARAPROFESSIONAL MATH GUIDEBOOK

LA MESA-SPRING VALLEY SCHOOL DISTRICT

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PARAPROFESSIONAL MATH GUIDEBOOK

Selection - In-Service Workshop - and Evaluation Overview

Involving the

Students
Paraprofessionals
In-Service
Records
Instruction
Teachers

of "76"

1973 1976

Prepared by

Dr. Albert J. Renaud - Project Manager

Sponsored by ESEA TITLE IV, PART C PROJECT

LA MESA-SPRING VALLEY SCHOOL DISTRICT

1976

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MATH GUIDEBOOK COMMITTEE

Cal Strickland	Principal
Tom Klipp	Teacher
Lori Poma	Teacher
June Nichols	Teacher
Kathy Price	Math Paraprofessional
Keith Meredith	Research Consultant
Donna Johnson	Math Consultant

MATH PARAPROFESSIONAL CONTRIBUTORS FROM PROJECT SCHOOLS

Lemon Avenue:

Kathy Katsoulas
Dorothy DeJoseph
Kathryn Price

La Mesa Dale:

Phyllis McDonald
Cynthia Campbell
Adele Fessenden

Rolando:

Arlys Smedsrud
Lilia Wood
Jennifer Oskamp

Maryland Avenue:

Patricia Harris
Sharon Smith
Louise Ford

La Mesa Junior High:

Margie Mitchell
Marilee Saunders

Parkway Junior High:

Dorothy Eagle
Jacqueline Ballbach

Typist:

Marge Ward

Artist:

Bill Burgin

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Chapter 1

Introduction

TEACHER
AIDE

INTRODUCTION

The United States is currently developing a rapidly growing body of school employees -- the classroom paraprofessional. Nearly 250,000 are presently employed in public schools and the National Congress of Parents and Teachers project a 1.5 million figure by 1977. Exact figures are not available for California; however, the California Teacher's Association in 1971 reported that at least one-third of all school districts are using some type of classroom aide. Federal, state, and local funds supporting the paraprofessional in California can be estimated in millions of dollars. In 1971-1972 the La Mesa-Spring Valley School District expended approximately $250,000 for Teacher-Aide salaries. The amount of $377,778 for Teacher-Aide salaries in the school district for 1973-1974 confirms the prediction of the National Congress of Parents and Teachers that Teacher-Aides will continue to increase in numbers.

In spite of the effort to improve instruction through the use of paraprofessionals, educators have gathered little data related to the effect of such educational assistance on student achievement and attitude in mathematics. Through ESEA, TITLE III Funding, La Mesa-Spring Valley School District participated in a three year research study (starting in 1973 and continuing through 1976) investigating the effect of paraprofessionals on student learning in the classroom.

Involvement of classroom paraprofessionals in the La Mesa-Spring Valley School District was focused specifically in the area of mathematics to observe if changes would occur in both student math achievement and attitude toward mathematics.

Four elementary schools participated in the project and involved students in 4th, 5th, and 6th grades and in two junior high schools involved students in the 7th grade. Over 1200 students participated in the study.

After the paraprofessionals were hired, they were randomly assigned to the trained or untrained category. Each was then randomly assigned to a selected school and grade level. Thus, the Project comprised three major components.

1. Classrooms with trained paraprofessionals who received math instruction through several in-service workshops extending throughout the school year.
2. Classrooms with untrained paraprofessionals not receiving any type of in-service training.
3. Classrooms with no math paraprofessionals at all.

The major emphasis regarding the effectiveness of the paraprofessionals on student achievement in math centered on the four objectives listed below:

1. Students in classrooms with trained paraprofessionals will demonstrate greater achievement gains in mathematics than will students in classrooms with untrained paraprofessionals.

2. Students in classrooms with untrained paraprofessionals will demonstrate greater achievement gains in mathematics than will students in classrooms with no paraprofessional assistance.

3. Students in classrooms with trained paraprofessionals will demonstrate greater attitudinal gains in mathematics than will students in classrooms with untrained paraprofessionals.

4. Students in classrooms with untrained paraprofessionals will demonstrate greater attitudinal gains in mathematics than will students in classrooms with no paraprofessional assistance.

This study included pre and posttesting of both attitude and achievement for all students involved in the Project.

SUMMARY

Continued growth and involvement of paraprofessionals in school systems throughout the United States requires us to look closely at the use of the paraprofessional as they confront the students of today and of the future. It may be that paraprofessionals, with specific training and knowledge of children, could be as indispensable in schools as are nurses' aides in hospitals. Currently there are colleges and universities offering courses for paraprofessionals in the area of Math, Language Arts, and other subject areas.

This Guidebook was written to share with others what has been learned in the three year math paraprofessional study involving students from 4th through 7th grade. This Guidebook is also intended for use by school districts that wish to initiate a particular type of paraprofessional program as aides continue to flow at an accelerating rate within the educational system.

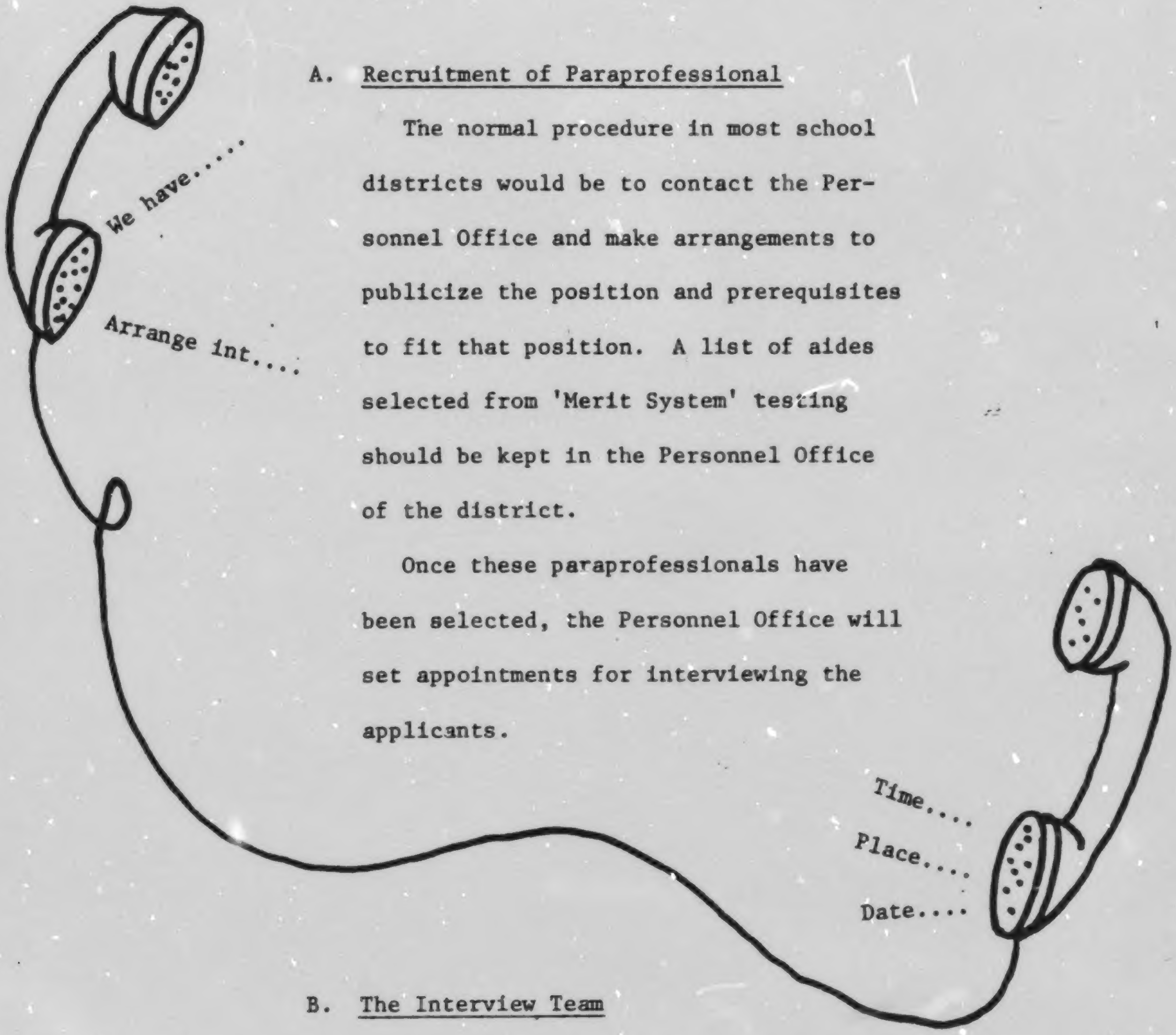
Chapter 2

Selection of Paraprofessional

MATH
AIDE
IS....

SELECTION OF PARAPROFESSIONAL

After a school or school district recognizes its need for a paraprofessional program, the next step is to select the correct type of paraprofessional to fit the needs of the program. For purposes of this guidebook, the major concentration will be in the area of mathematics.


We have.....
Arrange int....
Time....
Place....
Date....

A. Recruitment of Paraprofessional

The normal procedure in most school districts would be to contact the Personnel Office and make arrangements to publicize the position and prerequisites to fit that position. A list of aides selected from 'Merit System' testing should be kept in the Personnel Office of the district.

Once these paraprofessionals have been selected, the Personnel Office will set appointments for interviewing the applicants.

B. The Interview Team

The interview team should consist of the following people:

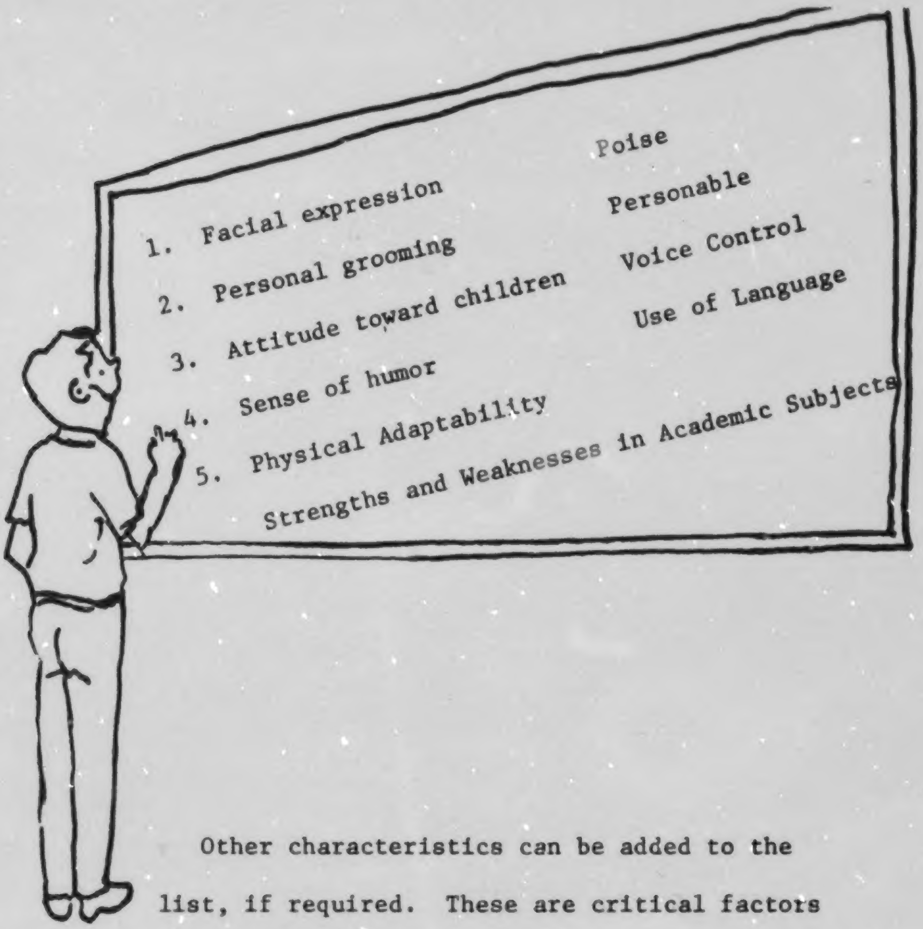
PROJ. MANAGER
TEACHER
PRINCIPAL

This will enable the applicant to receive an evaluation that is honest and fair and that the evaluation team can then make a rational judgment about the applicant's ability to fit that particular position in a constructive and meaningful manner.

C. What the Interview Team Should Be Looking For

One of the first tasks for the interview team would be to review the applicant's formal application form as shown on page 9 which includes pertinent information regarding the applicant's background. The team can then approach the interview with better understanding and be prepared to make the applicant feel at ease throughout the interview period.

As the team proceeds through the interview with the applicant, important factors should be noticed, such as:


1. Facial expression
2. Personal grooming
3. Attitude toward children
4. Sense of humor
5. Physical Adaptability
Poise
Personable
Voice Control
Use of Language
Strengths and Weaknesses in Academic Subjects

Other characteristics can be added to the list, if required. These are critical factors of prime importance for persons who will be working in schools and have direct contact with teachers and children.

D. Interview Evaluation Form

An important component in the selection of paraprofessionals is the evaluation form (page 11) which is completed after the interview. Each member of the team evaluates the applicant. After the form is completed, an appraisal score is recorded. This procedure provides the team with another type of instrument to determine the applicant's suitability for the position.

LA MESA-SPRING VALLEY SCHOOL DISTRICT
4750 Date Avenue
La Mesa, California 92041

APPLICATION
CLASSIFIED
EMPLOYEES

For Office Use
Test: ____________
____
Date: ____________
____
List: # ____________
FW ______ FAW ______

NAME: Mr.
Mrs.
Miss. ______________________________
(last) (first) (middle)

ADDRESS: ______________________________
(street) (city) (zip code)

SOCIAL SECURITY NO. ____ - ____ - ____

TELEPHONE NUMBER: ____________ ____________
(home) (business)

BIRTHDATE: ______________
(mo.)(day)(yr.)

Height ____ ft. ____ in. Weight ______ lbs.

WILL YOU ACCEPT:

________	________	________	________	________
Full-time Employment	Part-time Employment	Substitute Employment	Temporary Employment	10 or 11 Month Employment

LIST POSITIONS FOR WHICH YOU ARE APPLYING: (Note: Use exact titles as listed in job description book)

1. ______________________ 2. ______________________

3. ______________________ 4. ______________________

Answer the following questions by checking Yes or No columns and complete additional information where applicable:

	Yes	No		Yes	No
1. Are you a citizen of the U.S.?			5. Have you previously worked for this District? If yes, complete the following: Year Left: ____________ Position Held: ____________		
2. Do you claim Veteran's Preference Points? If yes, complete the following: Branch of Service ____________ Date Entered ____________ Date Released ____________ Type of Discharge ____________			6. Have you had any serious illness or operations during the past 2 years? If yes, please explain: ____________		
3. Do you claim Veteran's Disability?					
4. Have you ever been convicted in a court of law for a misdemeanor or felony? If yes, explain: ____________			7. Do you have any physical defects? If yes, please explain: ____________		

EDUCATION: Circle the highest grade completed:

Elementary or High School: 6 7 8 9 10 11 12

College: 1 2 3 4

Name and Location of last Elementary or High School attended: ____________

Year left or Graduated: ____________

(continued on reverse side)

EDUCATION (Continued)

List any higher education such as Colleges, Universities, Business or Trade Schools or Special Training:

Name of School	Years Attended	Degree or Certificate

Do you type? __________ If yes, list approximate speed ____________________

Do you take shorthand? ____________ If yes, list approximate speed ____________________

EMPLOYMENT RECORD: (Begin with your PRESENT or last position held and work back. Account for all employment for the past ten years).

Present or last position

Employer: ______________________________ Dates Employed ________ ________ From To

Address: ______________________________ Salary per Month: $ ________

Title of Position: ______________________________

Duties Performed: ______________________________

____

______________________________ Reason for Leaving ______________

Employer: ______________________________ Dates Employed ________ ________ From To

Address: ______________________________ Salary per Month: $ ________

Title of Position: ______________________________

Duties Performed: ______________________________

____

______________________________ Reason for Leaving ______________

Employer: ______________________________ Dates Employed ________ ________ From To

Address: ______________________________ Salary per Month: $ ________

Title of Position: ______________________________

Duties Performed: ______________________________

____

______________________________ Reason for Leaving ______________

COMMENTS: (Use this space for additional employment information or information pertinent to your qualifications).

____

____

____

Please check your completed application form to see that you have answered all questions.

READ CAREFULLY BEFORE SIGNING:

I hereby certify that all statements made in this application are true, and complete, and that any misstatement of material facts will subject me to disqualification or dismissal.

DATE: ____________________ Signature ______________________________

La Mesa-Spring Valley School District
9/1/72

QUALIFYING INTERVIEW EVALUATION FOR CLASSIFIED EMPLOYEES

NAME OF CANDIDATE ______________________ Interview for ______________________
Position Title

FACTOR		REMARKS	Outstanding	Above Average	Average	Marginal	Unacceptable
1. Appearance	Poise, Grooming, Attire						
2. Personality and Self-Expression	Maturity, Manner, Emotional Stability, Speech Facility, Adaptability						
3. Judgment and Alertness	Analytical Ability, Decisiveness, Receptivity to Ideas and Reaction, Logical Response						
4. Attitude Interest and Dependability	Job Insight, Philosophy, Stability, Self-Reliance, Character, Reliability						
5. Physical Factors	Apparent health and vitality, mentally and physically competent to perform duties						
6. Training	Relevance, Amount, Quality, Candidate's success						
7. Experience	Relevance, Level of Responsibility, Special knowledge applicable to position						

Appraisal Score	**

**RATING SCALE (Percentile)

Under 60	Unacceptable (See Remarks)
61 - 70	Marginal (Qualifications are insufficient)
71 - 80	Average
81 - 90	Above Average
91 -100	Outstanding

DATE ______________

Signature of Interviewer ______________________

Instructions: Place a check mark in one appropriate box for each of the seven items listed. Determine from the Rating Scale candidates area of standing against established requirements for position and qualifications. Enter this percentage score in Appraisal Score box.

_______ Selected for position

_______ To be considered later _____ Unacceptable

COMPLETE THE FOLLOWING INFORMATION FOR SELECTED CANDIDATE ONLY:

Beginning Date ______________________

Ending Date for School Year ______________________

Title of Project or Program if Federally Funded or State Funded (also Staff Pilot Projects)

____

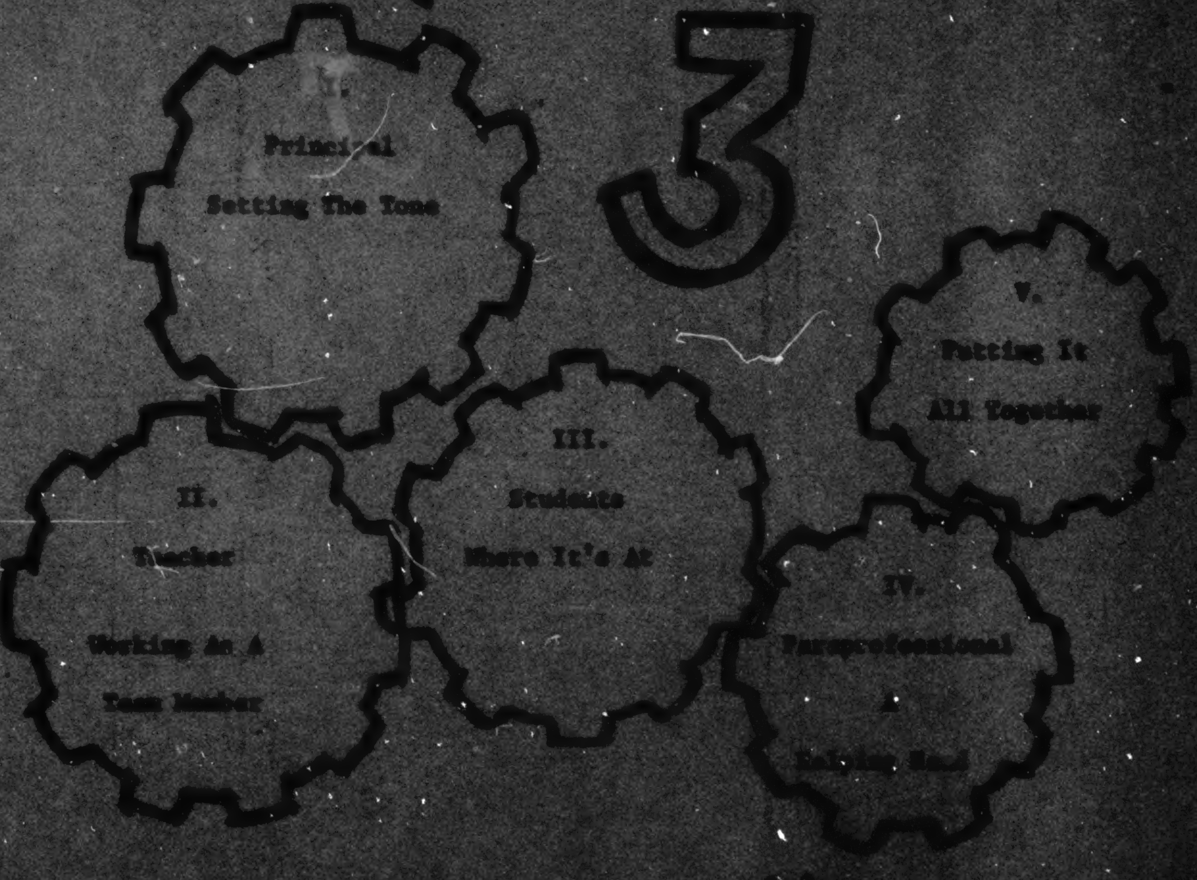
Personnel Division
Revised 9/72

E. After the Interview

Evaluation of the applicant must be in terms of the specific needs of the school and the experience and skill of the applicant. Applicants not chosen for employment should receive a letter informing them of the Team's decision, indicating that their continued interest in the school district is important and other positions may be available at a later date.

We appreciated your coming
It was my pleasure

Chapter 3


I.
Principal
Setting The Tone
II.
Teacher
Working As A
Team Member
III.
Students
Where It's At
IV.
Paraprofessional
A
Helping Hand
V.
Putting It
All Together

Working Together

Principal -- Setting The Tone

As the educational leader of the school, the principal not only assumes the responsibility for administering the school program, but also must accept the responsibility for setting the proper tone for developing harmonious working relationships among the various members of the school staff. This administrative support is crucial to the success of a paraprofessional program. To insure its success, the principal must believe in, demonstrate a commitment for, and be actively involved in the paraprofessional program.

First, the principal must view the paraprofessional as a valuable member of the instructional team -- one who can make possible a greater degree of flexibility and individualization of instruction.

Second, the principal must demonstrate a full commitment of support in order for a paraprofessional program to be effective and have value in serving the educational needs of children.

Third, the principal must be actively involved in the operations of the paraprofessional program. Some suggested ways are as follows:

> Welcome the paraprofessionals in such a way that they would feel a part of the 'team.'
>
> Review school operations and policy with the paraprofessionals at periodic intervals. Changes do occur and the paraprofessionals should be made aware of them.
>
> Support in-service programs for the paraprofessionals as they work to develop new techniques for carrying out their responsibilities in the classroom.
>
> Be aware of individual differences in the training and background of the paraprofessionals and proceed accordingly. Paraprofessionals who have been trained through in-service workshops should be more effective performing their duties than those who have not been trained.
>
> Be alert for problems that may arise in the school, whether it be in the classroom, on the playground, or in the faculty lounge. Alertness and quick response can avoid serious problems.

Encourage teachers to use the paraprofessionals in a variety of ways, thus increasing their effectiveness in serving the needs of children.

Invite paraprofessionals to staff meetings. This indicates that they are an important part of the 'team.' Their suggestions are of value to the overall school program.

The Teacher and the Paraprofessional -- Working as a Team

At last! A Paraprofessional!

The paraprofessional can be a boon to the classroom teacher. The primary concern, of course, will be how to make the experience beneficial for the pupils, the aide, and the teacher.

Responsibility of the Paraprofessional

The classroom teacher should be sure that the paraprofessional is conscious of responsibility to the pupils, the teacher, the school, and the community. The paraprofessional may assume some of the duties of a certificated employee, and must, therefore, understand the legal status as a paraprofessional and the role in discipline situations. A training or pre-work session for all paraprofessionals could deal with this, thus relieving the teacher of the responsibility.

Preparing for a Paraprofessional

Have a pamphlet prepared for the teacher to assist in preparing for a paraprofessional. The teacher needs to set the tone and needs to know what is expected and the legal aspects. Suggested ways to use the paraprofessional might be useful. This information will help relieve anxiety concerning particpation in a paraprofessional program.

Working with a Paraprofessional

Teachers should be appraised of all relevant information about Paraprofessionals before they are assigned. They should then have a voice in these assignments. In working with the Paraprofessional, the teacher should:

1. Provide adequate time where paraprofessional skills can be developed.

2. Paraprofessional should be made aware of the various tasks that are to be performed and reasons for carrying out those tasks.

3. Make directions very clear so the paraprofessional understands what is expected.

4. Trust is important when working with a paraprofessional.

5. An attribute by the teacher is one of having a patient hand when working with a paraprofessional.

6. Make corrections (when they are necessary) in private -- never in front of the children.

7. Strive constantly with the paraprofessional to improve instructional climate in class.

8. Take the opportunity for additional individualized instruction since having the paraprofessional can provide this time for you.

9. Continually evaluate the contribution of the paraprofessional in terms of improving instruction. This should be done cooperatively with the paraprofessional.

Fundamental to efficient use of a paraprofessional is a cordial relationship between the teacher and the paraprofessional. There must be an understanding of their separate and coordinate roles. Remember -- this is a team. The teacher and paraprofessional pull together in order to improve math instruction for all their students. Both should strive for a long term association. Much is to be gained by this commitment of two members working well together on a team.

Using the Services of the Paraprofessional

The teaching style will probably be the foremost factor determining how the paraprofessional will be used. If the teacher uses groups, the paraprofessional may be responsible for one group; if the teacher keeps the class together for a total project, the paraprofessional may be there to assist or help a small group of learners with special needs. If the teacher uses an individualized approach, the paraprofessional will be invaluable to assist and teach individuals needing help; when the teacher uses a combination of these or other methods, the paraprofessional's duties would alter and change accordingly.

Some specific areas of assistance might be:

- Presenting story problems.
- Presenting drill games.
- Checking papers with pupils.
- Supervising seatwork periods.
- Working with a small group in a basic concept.
- Researching books for drill material and making work sheets.
- Working with groups for advanced or remedial work.
- Making charts.
- Developing and putting up bulletin boards.
- Correcting drill sheets so they can be returned the same day.
- Using manipulative objects with slow learners to reinforce the teaching of basic facts.
- Developing, making, and teaching games to drill certain functions.
- Recording math progress.
- Assist supervising tests.
- Become acquainted with math materials in the audio-visual center and help children learn to use them.
- Make telephone calls to mothers to procure their help in grading papers, and thus free the teacher and aide for more instruction time.
- Tape recording math drills.
- Using flash cards for review.

-- and many more. Each teacher could add to this list.

Now to Begin!

First, determine how the paraprofessional is to be used and what areas are to be covered. The personality and the talents of the paraprofessional and the personality and working style of the teacher will form the basis. A few days must be allowed for the teacher to observe the paraprofessional's abilities and temperament and time to get acquainted with the pupils and the situation. During this time, various tasks may be assigned.

In the beginning, let the paraprofessional observe, roam around the room, get to know the childrens' names, and help those individuals who seem to need a 'helping hand.' Assist the paraprofessional to observe the class as a whole and notice those who are not attending to their work by playing, daydreaming, talking, etc. Note those who are hesitant and not seeming to cope with the problems. Let the paraprofessional work with a small group to 'get the feel' and build self-confidence. The paraprofessional should do some of the bookwork and recording -- this relieves the teacher and gets the paraprofessional more involved.

As these tasks are mastered a little at a time, the teacher will find out which are most beneficial and meet the greatest need for the program.

Commitment and Lesson Plans

The total commitment or involvement of the paraprofessional is determined by each case. Some teachers may: turn over to the paraprofessional entire blocks of time, make them responsible for certain groups, allow them the responsibility for certain concepts, or give them the management of a certain project. This must be determined by the teacher's program, the proficiency of the paraprofessional, plus the skill of the teacher in planning.

Lesson plans would be involved in any of the above activities. The activities would be determined by the desire of the teacher and the abilities of the paraprofessional. It is conceivable that the paraprofessional could make plans for teaching a group or a concept. In this case, plans would need to be written out by the paraprofessional and submitted to the teacher beforehand for approval and/or suggestions. It is also possible that plans could be outlined together during a conference period. Some teachers may prefer not to use either of these ideas, but prefer to plan and assign tasks from them to the paraprofessional.

Conferencing with the Paraprofessional

Time should be allowed occasionally for the teacher and paraprofessional to talk and make plans. Johnny's particular needs or his behavior resulting from learning or home problems should be shared to allow for better understanding and revised teaching techniques. The teacher's wider experience in most cases, if shared with the paraprofessional, will allow the paraprofessional to become more attuned and a more sensitive helper. Talk about specific students, and ask the paraprofessional's interpretation of situations. The discussion of the teacher's goals and ultimate objectives will help the paraprofessional to formulate ideas and procedures. On the other hand, the teacher will sometimes gain from the fresh approach the paraprofessional brings and the recent training that has been acquired. Discuss also the rapport between a young paraprofessional and the children. Discuss how it can be utilized to gain a cooperative attitude in the students. Help the paraprofessional avoid the pitfalls of 'being a pal.' Show or indicate when you are pleased. A pat on the back is the best incentive for good performance.

May All Your Math Periods be Happy Ones!

Give the paraprofessional responsiblity. Allow deviations from a pattern within the areas assigned. Keep up a good rapport with one another. It will pay off magnificently in growth for all concerned.

Student Statements Regarding Involvement of Math Paraprofessionals

Students were asked questions regarding their attitudes toward math and the difference an aide makes in a classroom/math environment.

When students were asked if they look forward to the math period or the math activities of each day, 51% said yes, and 16% felt they sometimes looked forward to math periods. Activities were then classified as math games, worksheets, working at the blackboard, etc., and 90% said they liked doing them. A further definition of Game Day brought a 95% positive response. It was also found that 51% of the students are made happy by the math activities, but an additional 30% had mixed feelings or fluctuated between being happy and being bored.

Fifty-five percent of the students felt that they had looked forward to math when school began, with 10% more fearful that it may be too hard for them. Halfway through the school year, feelings had improved with 80% liking math, but 10% had verified that math was hard, but that the situation was improving and they were hopeful their final results would be favorable.

Seventy-five percent of the children are not embarrassed about asking for help, but an additional 14% are not certain they get help when they need it, which makes them hesitate to ask for help. However, the children are aware that there are many people for the teacher and the paraprofessional to help.

On focusing attention to the paraprofessionals, 87% of the students feel at ease asking the paraprofessional for help, 92% feel the paraprofessional has a good attitude about helping them, and 87% of the children questioned would rather be in a classroom that has a math paraprofessional.

The children verbalized many of their feelings and opinions:

"I thought when school started I was going to learn a lot, but I don't think we've been doing hard enough stuff."

"I thought math was going to be harder than last year .... it is harder."

"The aide helps us get two different ways of instruction."

"The teacher can't help everyone at the same time."

"I am embarrassed to ask questions because in front of the class you feel dumb, like you don't know anything."

"I don't like asking the aide for help because I want to do it myself but I just can't figure out how."

"I am happy about math but if we did not have our math helper I would be bored."

"The math aide helps me when I don't understand the teacher."

"The aide helps me with a lot of patience."

"The aide is always smiling and being cheerful."

"She gives us other situations with the same kind of problem."

"I like the way my aide tells us a new way to do math."

"The aide helps me feel more secure."

"The aide helps me very carefully."

"She discusses it with me."

"They have a way of making math fun."

"She really knows math and she knows how to explain the work so kids can understand it."

"I would rather try to figure out the problem by myself, but when I'm stumped it's a good thing to have a math aide."

"Math aides are not crabby and don't yell."

"Games let you relax."

"Games give us a chance to have fun with math."

"Games let you learn to use math in different ways."

"Games give you a rest from other math."

"I do not like math much, but it's easier when a math aide can help you."

"She describes things but doesn't tell you the answer and that's good and bad."

"She doesn't tell me the answer, she really helps me."

"Having a math aide is a lot better than just one teacher and I like asking the math aide more than the teacher because I feel funny asking the teacher and feel a lot better asking the aide because it seems she understands more of the way we think."

"I could not get along without the math aide."

"If we didn't have an aide, it would be hard."

"I don't have to wait so long to get help."

"I know the aide will not make fun of me if I ask a question."

"I can ask questions because other people have to ask questions too."

"I am embarrassed sometimes because I feel stupid not knowing the answers."

"I wish we had math all day long."

Definition of Math Paraprofessional: A Helping Hand

A math paraprofessional is a teacher's aide whose purpose in the classroom is to stimulate thinking about math: children's thinking, and sometimes, teacher's thinking. A math paraprofessional needs to encourage children's confidence in math skills at whatever level the child is working. The paraprofessional needs the ability to focus on specific needs of the children so that each child's performance and understanding in math is at an acceptable level for his or her grade and ability. A math paraprofessional helps the teacher in teaching math and is in the classroom with the purpose of doing many of the jobs relating to helping children learn math concepts. The paraprofessional is in the classroom to encourage the child's self-image and interest in a specified subject: a 'you-can-do-it' attitude.

The term 'paraprofessional' has a feeling about it that is more constructive than 'Teacher's Aide.' A Teacher's Aide implies someone who is in the classroom to 'aid' the teacher. This person generally does not need special qualifications to run off papers, pass out necessary items, grade papers, help in clean-up, monitor children, or any of the tasks a teacher may designate to help his or her day proceed more smoothly. A trained aide, or the constructive use of the word 'paraprofessional,' also designates a helping hand in the classroom, but one with the added distinction of having special training. This person can be qualified to assist in a classroom because of a background in a specific subject, or career, or personal experience. The paraprofessional attends workshops that a teacher may not have time to attend and can bring valuable information into the classroom that would have been lost without the added advantage of that personal effort. The paraprofessional also works in a professional atmosphere and can make decisions regarding student competency, can prepare papers to instruct math, can do research to keep ideas fresh and new, can diagnose student math needs, and can create math games to expedite the learning process.

As every teacher is different, every teacher uses paraprofessionals differently. Sometimes teachers do not know how to use the paraprofessional. They may give them too much responsibility, such as leaving the paraprofessional in charge of a group of children not familiar to the paraprofessional; or they may not give them enough responsibility. The math paraprofessional/ teacher relationship needs understanding, and the ability to communicate, to facilitate the most useful classroom environment. It should be the paraprofessional's incentive and initiative that helps to create a working relationship between teacher and paraprofessional which will benefit students.

Trained paraprofessionals should be placed with teachers who know the paraprofessional's value and trust her ability to relate to children as individuals. Some teachers are not comfortable with paraprofessionals working with the children and these teachers should have a trained paraprofessional. A math paraprofessional does not need to be trained in math to be sent out of the classroom to do clerical work.

Paraprofessionals are assistants to a professional and should conduct themselves as such. They should be willing to give of themselves and their time; e.g., attend workshops offered that might be beneficial to the job or to the student's learning experiences, to be enthusiastic about the job, and be able to assist the professional in providing stimulus through games, activities, etc., to enrich student's math experiences.

Thus, a paraprofessional is someone working alongside a professional, one who is assisting, or taking the place of, a second professional in a classroom, qualified to search out and diagnose student needs in comprehending math concepts.

The position of paraprofessional needs clarification. As the need for trained paraprofessionals increases, the need for subject-related skills increases. A job classification involving specific requirements is needed which can be met by an untrained paraprofessional who wants to advance.

This relationship provides the basis for mutual understanding and respect for one another.

Personal Development Through Math Workshops

This math program uses trained and untrained paraprofessionals. The attitude and enthusiasm of the trained paraprofessionals changed from 'how do I use the materials and how do I use myself in a useful math environment,' to a workable knowledge of their positive use in the classroom. The workshops aided in development of understanding of math concepts and showed ways to teach concepts with a minimum of difficulty. Math can be fun.

One of the primary jobs was the use of the manipulative math materials. The untrained paraprofessional can learn how to use them also, but can they learn alternate ways to use them? The workshops also stimulated the trained paraprofessionals in developing their own materials, and additional use of manipulative materials.

In addition to the games, math procedures, and the use of the materials, paraprofessionals also learned how to get along with the students and how to effectively interact with children and teacher. Math is not just putting numbers on paper, but it is also helping a child learn how math is used.

Trained paraprofessionals are confident about bringing new ideas to teachers, and are always seeking out new math materials, make up math materials, and can think of ways to reach a child within a problem area, keep track of a child's skills and growth, and help children have confidence in their own knowledge and growth potential.

Chapter 4

PRE-SERVICE TRAINING

An Explanation

Teachers must have training and certification before they begin working. A valid assumption would seem that in order to become effective members of the instructional team, paraprofessionals also need some sort of training. There is need for basic pre-service training before the paraprofessional ever enters the classroom. Once this is concluded, the paraprofessional should have a basic degree of awareness and information necessary for functioning as a member of the school's instructional team.

The proficiency in a specific job situation, and role as a member of the school's instructional team, are developed through experience on the job and continuing in-service programs. However, prior to placement in the school, paraprofessionals should be familiar with their role and expectations, curriculum goals, materials, equipment, ethics, and other general information deemed a prerequisite to assuming an active role in the classroom with children. Following is a general outline around which a pre-service training program for paraprofessionals might be developed.

PRE-SERVICE TRAINING

Fundamental pre-service training should encompass:

A. Orientation to the schools.
B. The role of the paraprofessional.
C. Curriculum and support services.
D. Materials, supplies and equipment.
E. Curriculum goals, objectives and materials.
F. Subject matter content.

These topics provide a basic foundation of awareness and information for involving the paraprofessional as a member of the instructional team.

A. Orientation to the schools

A tour of the school site and introduction to the principal, teachers and other staff members should be provided. At this time a brief summary of routine school procedures such as check-in, time sheets, reporting absences, and basic expectations of staff members should be provided.

B. The Role of the Paraprofessional

What professional ethics are expected? How important are community relations? What is the role of the paraprofessional in working with the teaching staff? These and other questions should be answered in a pre-service session which might include topics such as:

1. Professional ethics.
2. Role as a part of the school staff.
3. Working with students.
4. Community and parent relations.
5. Do's and don'ts for paraprofessionals.
6. Legal Information.

C. Curriculum and Support Services

Pre-service training should make the paraprofessional aware of things such as support personnel, resource teachers, community resources, supplementary materials, and audio visual aids available through the district.

D. Materials, Supplies and Equipment

These may vary from school to school, but the paraprofessional must be aware of what is available. This portion of pre-service training should include:

1. Procedures for obtaining supplies.
2. Ordering audio visual materials and use of AV equipment.
3. Training in use of duplicator, thermofax and other workroom equipment.

E. Curriculum Goals, Objectives and Materials

An overview of the district's goals and objectives should be provided. Exposure to textbooks, diagnostic, prescriptive and evaluative materials and their place in the curriculum will create awareness of the total curricular scope and sequence. Consideration of the district's testing program, manner of grouping students, individualized instruction, and supplementary materials and texts will add to the paraprofessionals' awareness of the curriculum.

F. Subject Matter Content

In conjunction with general awareness of the curriculum, a basic understanding of subject matter content is necessary. This exposure to content should provide initial confidence and knowledge necessary for performing with competence.

SUMMARY

A basic pre-service training program should be provided for paraprofessionals. The purpose of the pre-service training is to create an awareness level which will serve as a foundation for the development of basic competency through experience and continuing in-service training.

Chapter 5

IN-SERVICE

Math
Workshop
Lessons

The pre-service and in-service lessons are conducted with these aims in mind:

1.) The paraprofessional's understanding of math topics in the school curriculum will be extended and strengthened.

2.) He/she will learn to use many materials other than the textbook in reinforcing specific mathematical ideas, and in remedial activities. (See Appendix G)

An outline of content follows. This is organized according to concepts as presented in the Strands Report, issued by the Mathematics Curriculum Committee of the State of California. Materials used in the development of these concepts is indicated in each section.

A. Numbers and Operations

1.) Addition, subtraction, multiplication and division of whole numbers.

Materials: Drill and Practice at the Problem Solving Level, Robert W. Wirtz, Curriculum Development Associates; Suite 414, 1211 Connecticut Ave.; Washington, D.C. 20036; Beans and Beansticks; Multi-base blocks.

2.) Fractions

a.) Concept of fraction,
b.) Equivalent fractions,
c.) Adding, subtracting, multiplying, dividing fractions.

Materials: Drill and Practice, Wirtz; Cuisenaire rods; Fraction Bars, Pattern Blocks, Unifix cubes; Tangrams; Discovery blocks.

3.) Place Value and Base

Materials: Multi-base blocks; Beansticks.

4.) Decimal Notation

Materials: Multi-base blocks.

5.) Percent

Material: Multi-base blocks.

6.) Pre-Algebra

a.) True, false, open statements, substitution.
b.) Development of arithmetic properties via identities. (Open statements which are true for any valid substitution.)

        c.) Function machines and formulae.
        d.) Cartesian graphing.
        e.) Signed numbers.

B. Geometry

1.) Concepts of polygonal shapes, names.

Materials: Pattern Blocks, Geoboards, Tiles.

2.) Congruent plane figures.

Materials: Pattern Blocks, Geoboards, Tangrams, Game of hexed.

3.) Similar figures

Materials: Pattern blocks, geoboards.

4.) Line and Point Symmetry

Materials: Tiles, Pattern blocks, geoboards.

5.) Concept of angle, angle measurement

Materials: Pattern blocks, tangrams, protractors, compasses.

6.) Properties of solid figures

Materials: Geoblocks, compasses, protractors.

C. Measurement

1.) Non-standard measurement.

Materials: Tiles, string.

2.) Metric system

Materials: Pan balance, balance scales, metric weights, metric volume jars, meter sticks, centimeter cubes, metric trundle wheel, metric geoblocks.

    a.) Development of standard measures, and meanings of milli-, centi-, kilo-.

3.) Scaling

Materials: Meter sticks.

4.) Development of perimeter, area, volume functions and formulas.

Materials: Cubes, Rods, Squares, Cuisenaire Company; Geoblocks; Geoboards; Tiles.

D. Logic

1.) Sorting and ordering.

    a.) Defining sets;
    b.) Development of notions of intersection and union.

Materials: Attribute blocks.

2.) Logic games.

Materials: Blocks, paper and pencil

E. Functions

1.) Deriving functions empirically.

Materials: Geoblocks; geoboards; pattern blocks; centimeter cubes; cuisenaire rods; Rods, Cubes and Squares; Tiles; Mathematical Balance; etc.

2.) Developing area and volume formulas via functions.

Materials: as under Measurement section.

3.) Cartesian graphing.

Materials: Graph paper.

4.) Multiplication Tables via functions and graphs.

Materials: Graph paper.

F. Probability and Statistics

1.) Gathering data, constructing charts.

2.) Interpreting charts - necessity of representative sample.

Materials: Graph paper, construction paper, scissors, glue, felt pens, etc.
American Wildlife Federation Booklet.

3.) Tasks in Probability.

Materials: Probability task cards from Good-Time Math,
Dice, Spinners, Cards, etc.

G. Problem-Solving

If we regard the mathematical tasks we accomplish as "experiments," then we are already approaching the concept of problem-solving. The idea of teaching fail-safe methods of problem-solving is self-contradictory. Approaching the ideas of mathematics in an experimental fashion encourages the student to draw from his previous understanding and his ability to see patterns and relationships. Thus, problem-solving, in our opinion, is not a topic to be taught in isolation, but throughout all the areas of mathematics, including computation.

Lesson I

I. Discussion of Strands Report by the Mathematics Curriculum Committee of the State of California, with particular emphasis on the idea that mathematics, even in our practical lives, is much more than arithmetical computation.

II. Numbers and Operations.

- A.) Introduction to book, Drill and Practice at the Problem-Solving Level, Wirtz.
    - 1.) Presentation of the author's philosophy that anyone capable of counting is capable of computation.
    - 2.) In-depth study of pages 1 through 27.

III. Geometry.

- A.) Designs with Pattern Blocks. Build larger shapes with small ones. During this activity, discuss names of polygons. In many instances, this leads to a clarification of names such as the fact that squares are rectangles, etc.
- B.) Find lines of symmetry by placing mirrors on designs. Discuss the meaning of symmetry.
- C.) Discuss the meaning of similarity by referring to larger and smaller squares, parallelograms, trapezoids, etc.
- D.) Discuss tasks which can be assigned to students. Look over commercially produced task cards.

IV. Statistics.

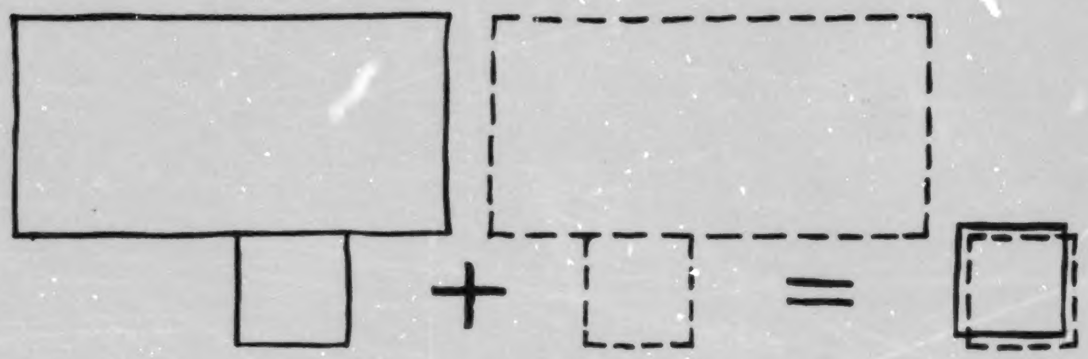
- A.) Look at some graphs previously constructed by students. Discuss the information contained in the graph.
- B.) Discuss some possible topics for graphing. Choose several. Collect data which will be brought to next class for the purpose of organizing and graphing.

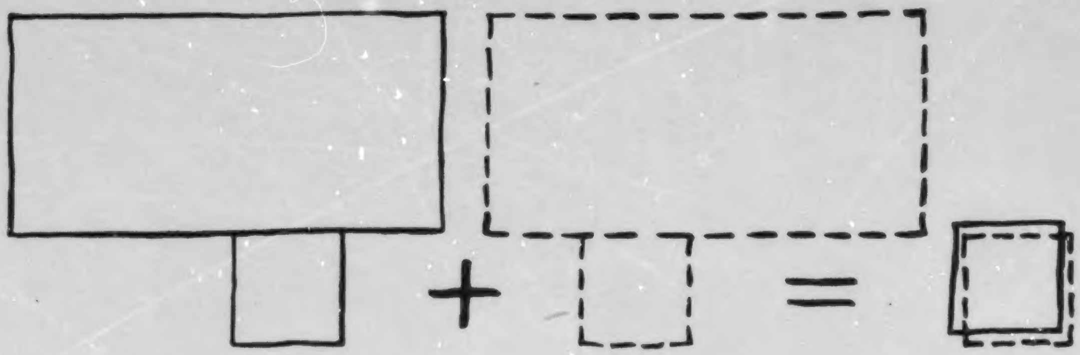
ADDITION PRACTICE

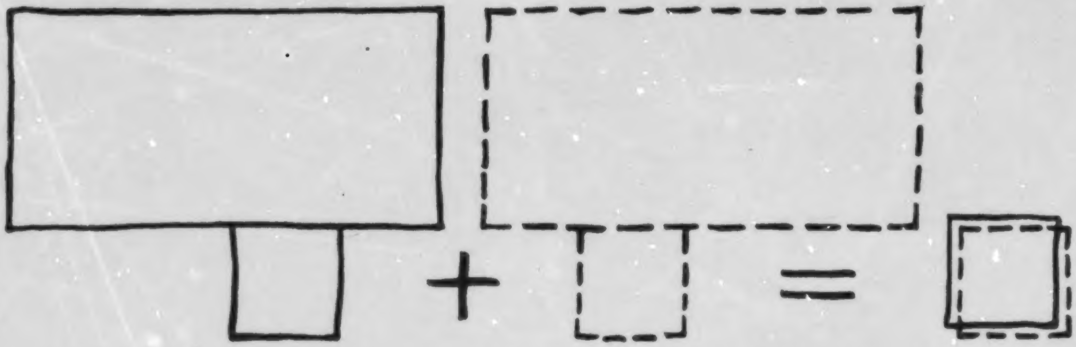
WHAT A TANGLE !

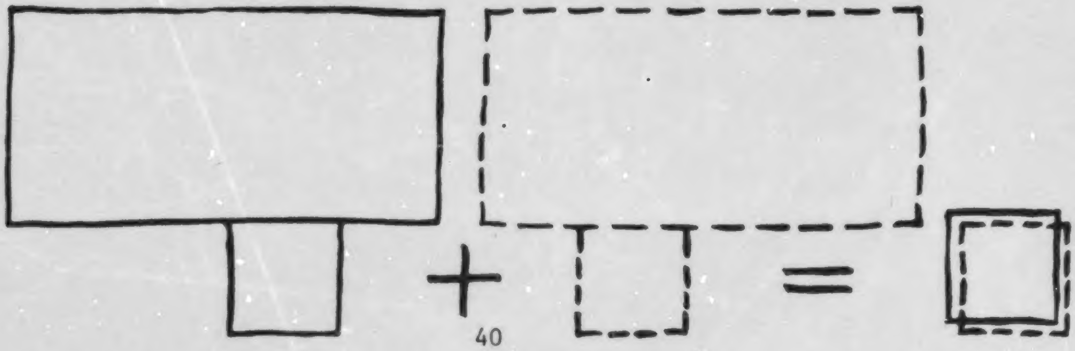
Can you start in the upper left hand corner of this number maze and draw a line connecting six numbers whose total is 30? Can you then start with the last number you use and connect five more squares to total 30 again? Can you continue using the final number in a group and five more until you have made seven groups each totaling 30? Can you do this without using any box more than once or ever crossing a line you have already drawn?............

5	8	4	3	5	6
8	4	3	5	9	4
9	4	6	6	3	7
7	3	2	7	5	5
4	6	5	6	4	8
4	7	4	1	3	3









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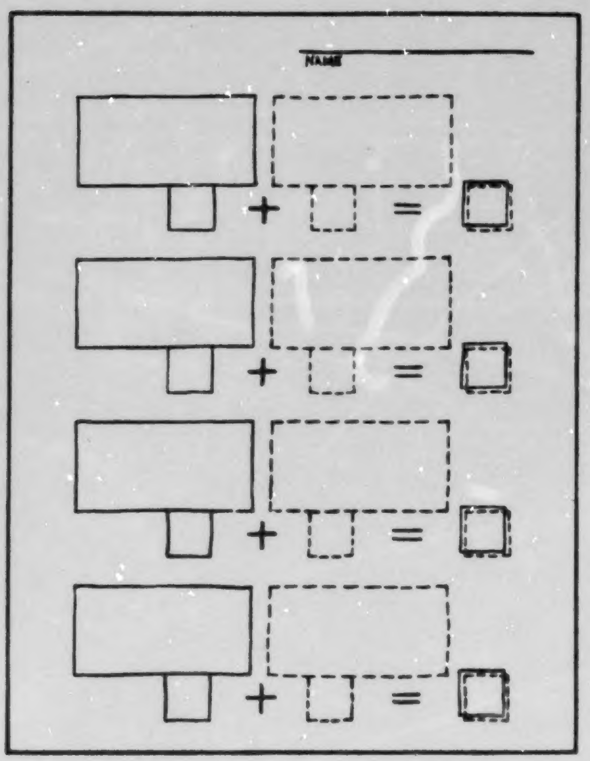
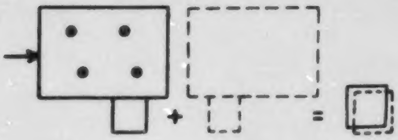


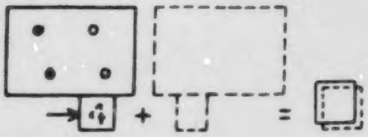
Children need counters.

This activity can be introduced as requests for a series of acts.

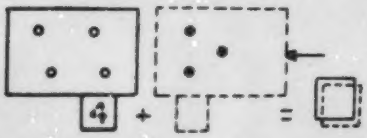
(1) Please put some counters in the large black-line frame.



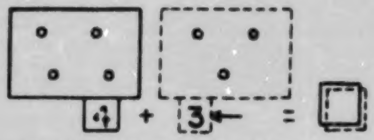
(2) Please write a number in the little square below that frame to show the number of counters in the frame.



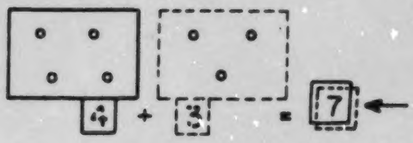
(3) Please put some counters in the other large frame . . . as many as you want.



(4) Please write a number in the little square below that frame showing the number of counters there are in the frame.



(5) Please find the total number of counters you have used in this experiment, and write that number in the double-line square on the right.

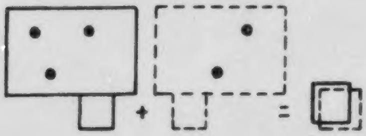


This final report can be read any way the child wants to: "4 and 3 makes 7" or "4 and 3 more is 7," etc.

As each child demonstrates he understands the procedure, he is ready to make up examples on his own —a completely individualized activity from that point on.

While we have considered 5 steps in the activity carried out in a given order, this is not intended as a prescription: rather, the procedure indicated demonstrates that, at no time, is the child confronted with more than one direction at a time and that he is asked to record the results of each separate count he carries out.

In fact, children very soon decide on their own sequence in making up their own examples. Usually they begin by putting counters in both frames.



and then fill in the number in each frame—and then find the total and record it.

As long as the reports are reasonable, we are unconcerned about the individual style of reaching conclusions.

We choose to refer to each example the beginner carries out as an "experiment" with things and to the "arithmetic" as his report of his experiment.

This page can also be particularized to specific experiments the learner is to carry out and report his outcome.

☐ × ☐ = ☐

☐ × ☐ = ☐

☐ × ☐ = ☐

☐ × ☐ = ☐

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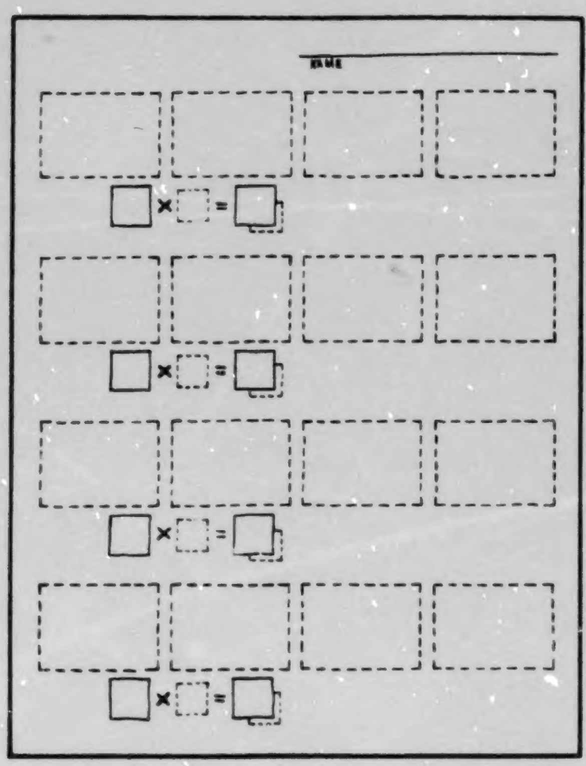
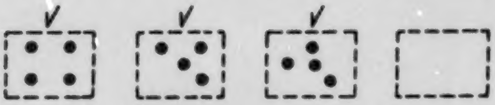


As the discussion proceeds, we will soon begin to use the former approach only, but please feel perfectly comfortable in adopting the latter sequence and language. Fortunately, the total will always be "12" regardless of how you interpret the statement:

$$\boxed{3} \times \boxed{4} = \boxed{12}$$

In these examples, the "number of boxes" is not indicated. When the child decides how many frames he wants to use, he checks that many or otherwise marks them.

A completed example would look like this:



$\boxed{4} \times \overset{\checkmark}{\boxed{3}} = \boxed{12}$ (our preference)

or

$\overset{\checkmark}{\boxed{3}} \times \boxed{4} = \boxed{12}$ (optional)

(We have included a small check above the small square with the number indicating the number of boxes—we use the second square; you may use the first square.)

What happens if we decide to put 7 counters in each of the "no boxes"?

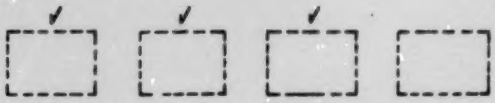
$$\boxed{7} \times \overset{\checkmark}{\boxed{0}} = \boxed{} \text{ or } \overset{\checkmark}{\boxed{0}} \times \boxed{7} = \boxed{}$$

If no frames are used, then there can be no counters in them—so we would write

$$\boxed{7} \times \overset{\checkmark}{\boxed{0}} = \boxed{0} \text{ or } \overset{\checkmark}{\boxed{0}} \times \boxed{7} = \boxed{0}$$

The total would be the same regardless of how many we chose to put in each frame.

Suppose we chose to put "no counters" in three frames.



$\boxed{0} \times \overset{\checkmark}{\boxed{3}} = \boxed{0}$

or

$\overset{\checkmark}{\boxed{3}} \times \boxed{0} = \boxed{0}$

If there are no counters in each of 3 frames, then there is a total of no counters—"0" counters.

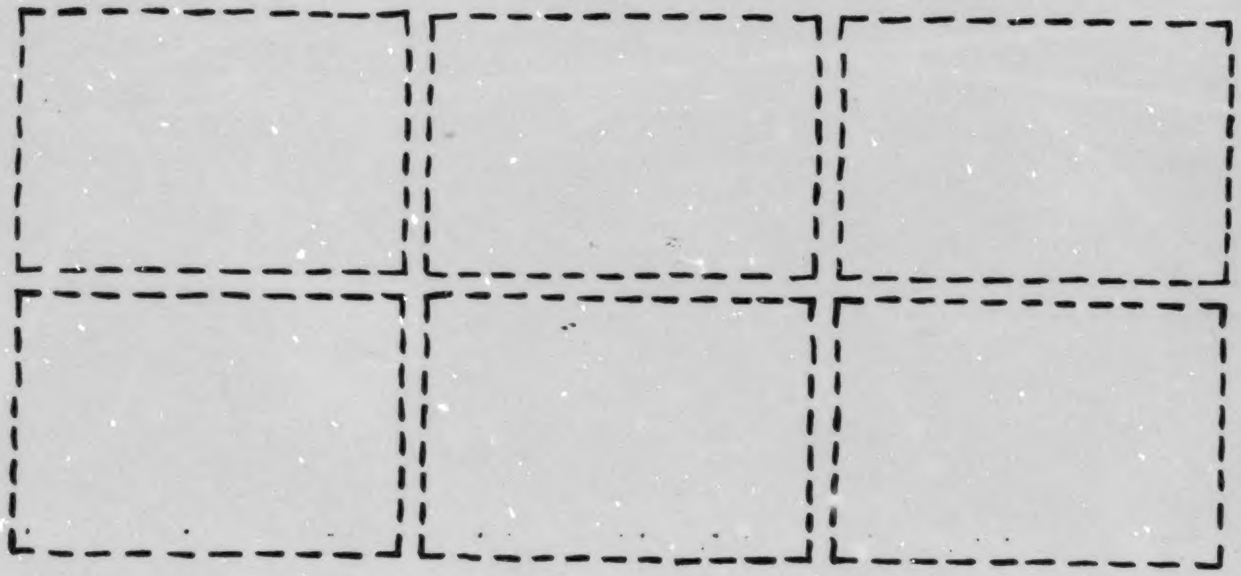
No other surprises will arise.

This form might also be used to report the results of all ways exactly 12 counters could be used in experiments (with only 4 frames available).

Or, the first two report-squares could be particularized to call for specific experiments.

$$\boxed{3} \times \boxed{2} = \boxed{}$$

____



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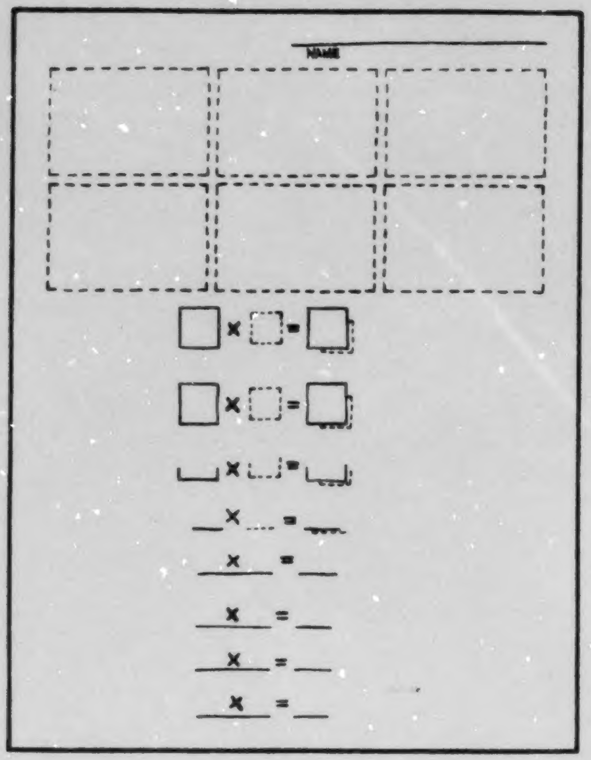

The number of frames provided has increased; and all the experiments are carried out in one group of frames. Further, the unique notation is phased into a standard form—

____ x ____ = ____

Many children will be finding other reliable ways to find the total number of counters used—the "product." Some will imagine the experiment and fill out the record knowing what the outcome would be. Others will use tally-marks—often an elaborate system so they can complete many examples in a single group of frames. Still others prefer using scratch paper in many different ways.

All methods that reliably produce sensible statements should be encouraged.

If any child wants to explain his system to others, and discuss it, fine! . . . since we are all involved in one great hunt for reliable shortcuts.

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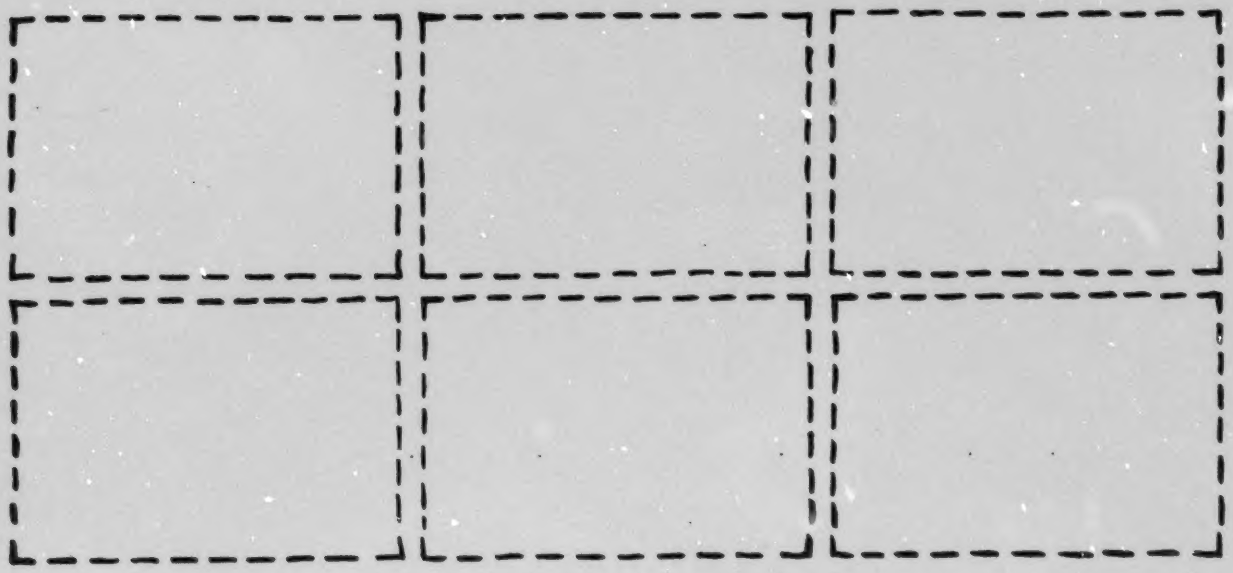

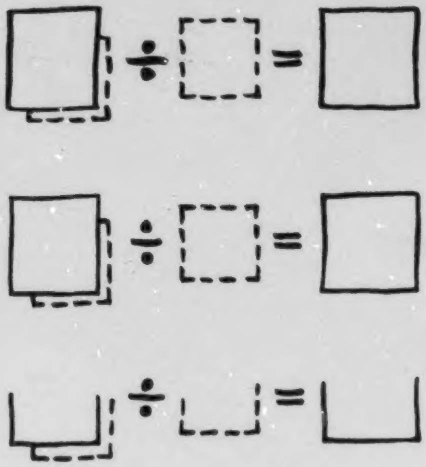

____ ÷ - - - - = ____

$\frac{\div}{\qquad\qquad\qquad} = \underline{\qquad}$

$\frac{\div}{\qquad\qquad\qquad} = \underline{\qquad}$

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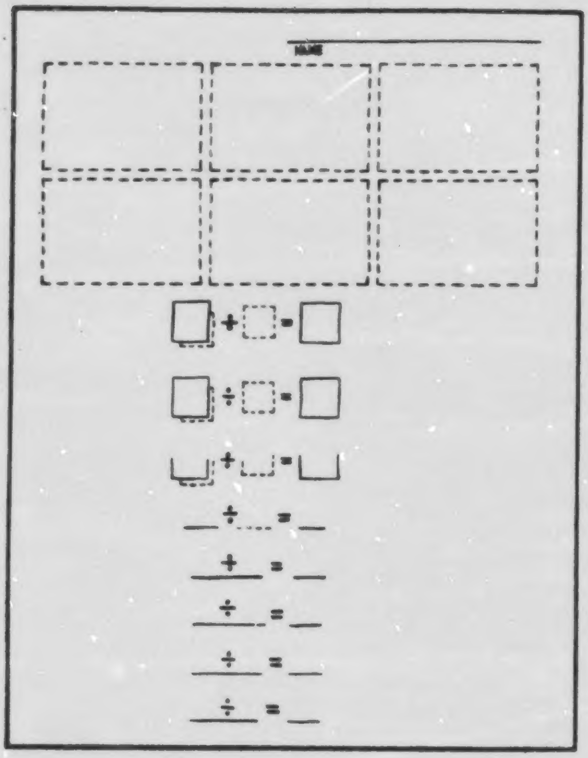
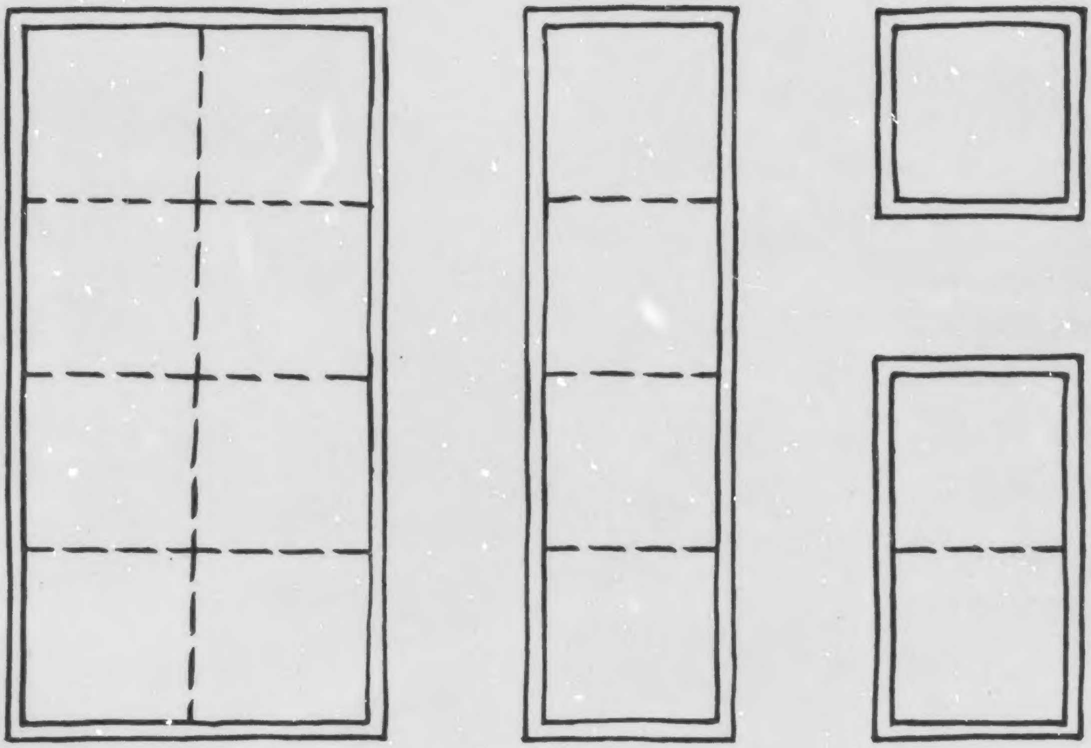


The options increase—6 frames are now provided. More experiments with a single group of frames. And the unique notation we have used phases into a standard notation.



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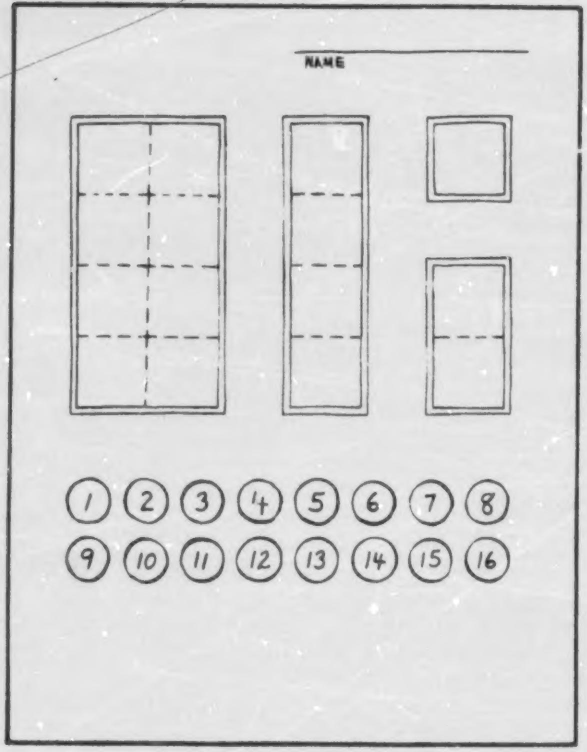


The name of this activity is "windows and panes." There are 4 "windows"—one with 1 "pane," another with 2 "panes" and others with 4 and 8 panes.

The children will need counters.

The rule is that if you put a counter on one pane in any window, you must put a counter on all panes in that window. Also, you must put only 1 counter on any pane.

The record is kept below:

✓ ① means "I can do it."

▨ means "I can't do it."

✓ ⑮ ⌇ ⑯ squiggle means "There aren't any more panes."

The completed record would show that there is a window or combination of windows for all numbers 1 thru 15—the total number of panes.

A special characteristic of this selection of windows with 1, 2, 4 and 8 panes is that there is a combination (or single window) for each number 1 thru 15—and only 1 combination. If you know the number of counters, then you know which windows are used.

A unique relationship in the series of windows can be demonstrated by having 2 children play a game of "one more." If Sally and Harry are playing and Sally goes first, her task is to put 1 counter on the sketch.

Then Harry puts on one more. He can do it only by moving Sally's counter.

Then Sally puts on a third counter—"one more," the name of the game—on the 1 pane windows.

Harry's job is to put on a fourth counter. He must move all three over to the 4-pane window and add a counter.

Sally adds one on the 1-pane window.

Harry moves that one down and adds one more.

Sally adds one in the 1-pane window.

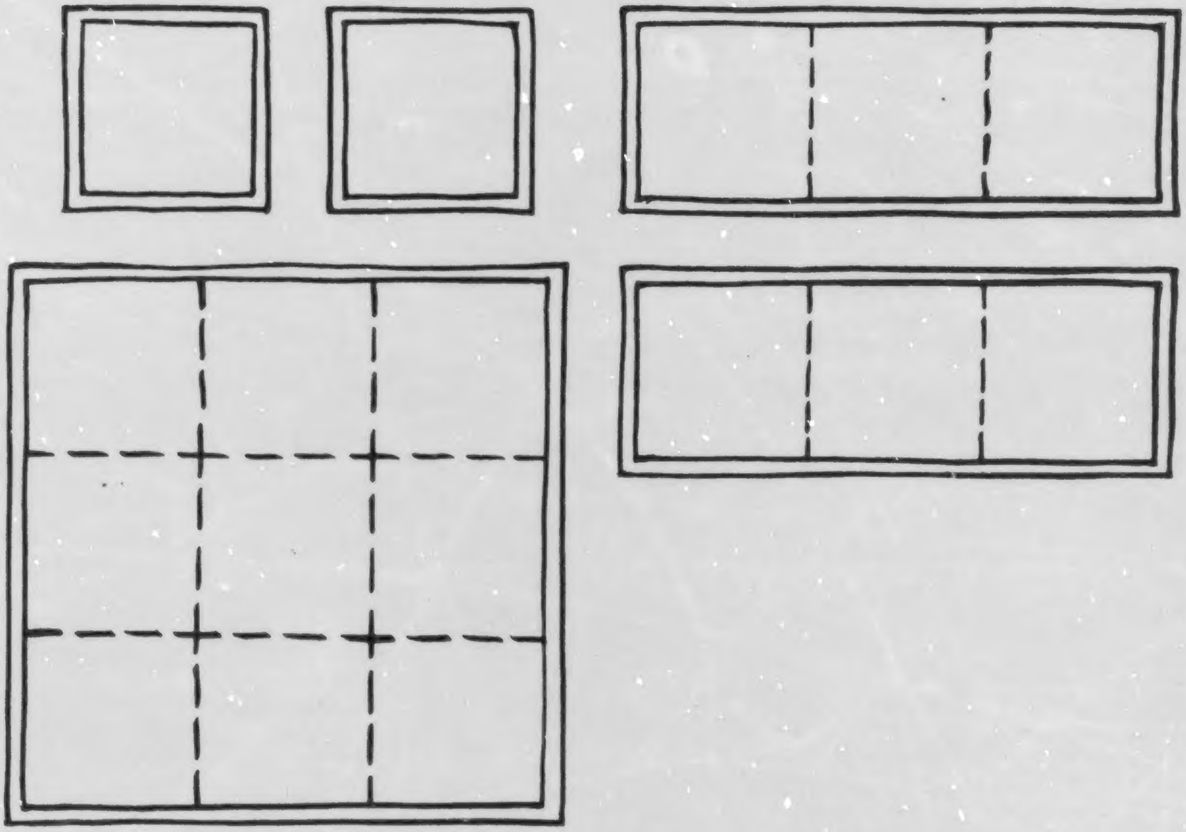
Harry's task: one more.

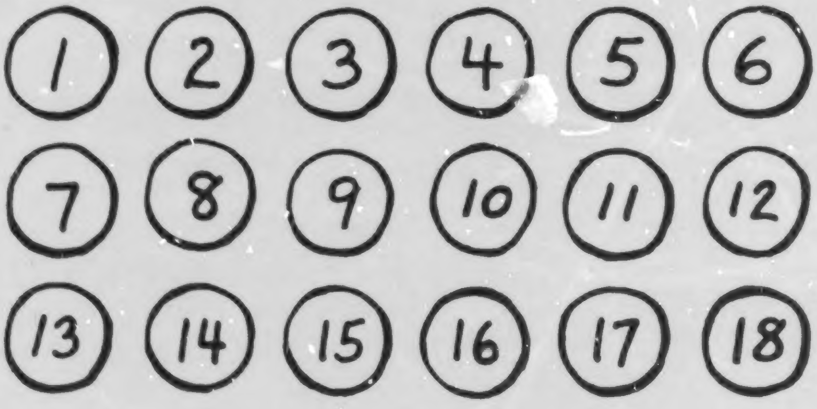
He must move all the counters to the large window and add one more for a total of 8.

About this time Harry has a strong feeling that he is doing all the work while Sally just keeps adding her counter to the 1-pane window.

(And everyone involved has a "first encounter" with the "binary system"—after 1, each larger number is twice the next small number:

1	2	4	8	16	
	(2 x 1)	(2 x 2)	(2 x 4)	(2 x 8)	etc.




1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18

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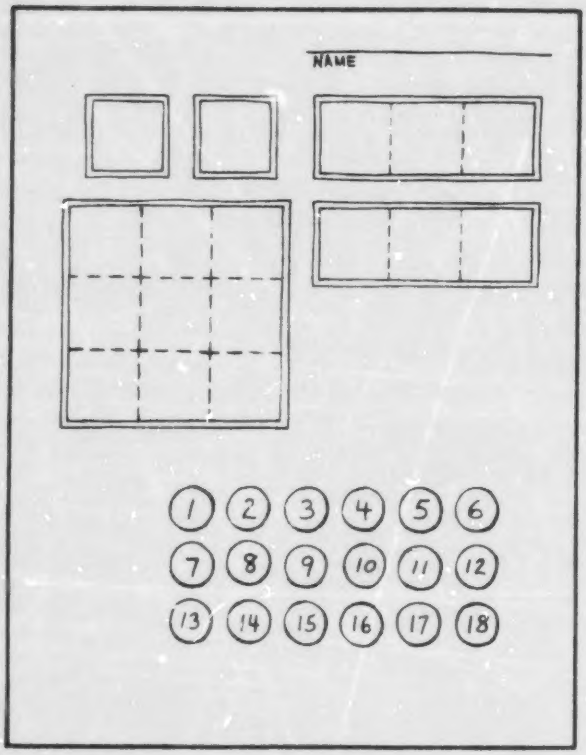


More "windows and panes."

The children need counters.

Again, the rule is: If you put a counter oh a "pane," you must put a counter on each pane in that "window" . . . and only one counter on any one pane.

There is a way to put on all counters 1 thru 17. As on the previous page, there is but a single way to put on any given number—except when there is an option to use either one of the 1-pane or 3-pane windows. Counters can be put on one or the other.

If Harry and Sally were to use this diagram for the game of "one more" described on the previous page, they would find a more equal division of labor.

They might persuade a third player to join the game—Al, perhaps. Sally plays first—on a 1-pane window, Harry plays next—adding one to the 1-pane window.

Al must move those over and add one to fill a 3-pane window.

Sally and Harry each put a counter on a 1-pane window.

Al must move them over to the open 3-pane window and add one more to fill it.

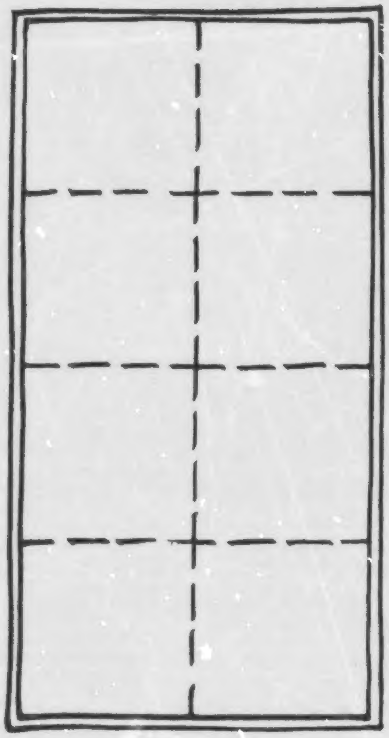
Sally and Harry each put a counter on a 1-pane window.

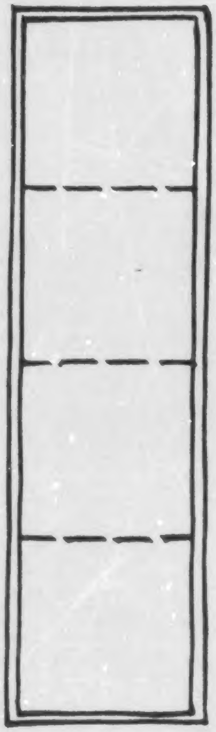
Al—well, Al is doing all of the moving of the counters.

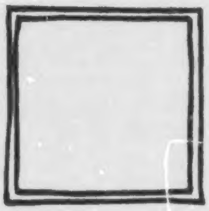
The selection of windows—1, 3, 9—starts with 1 and then each number is 3 times the next smaller number . . . and there are two windows of each kind:

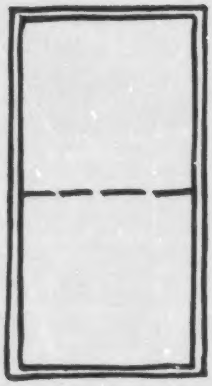
1	3	9	27	81
	(3x1)	(3x3)	(3x9)	(3x27)

____









8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐

8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐

8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐

8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐

8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐   8 4 2 1 _ _ _ _ ☐

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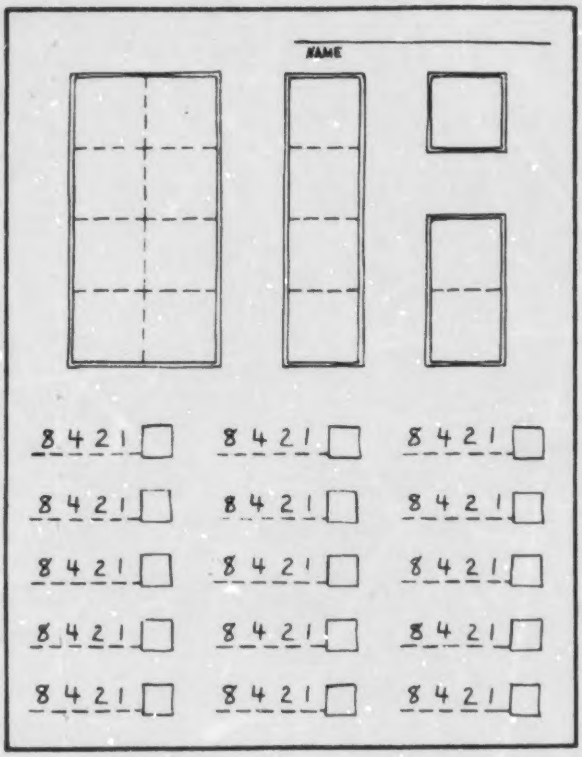
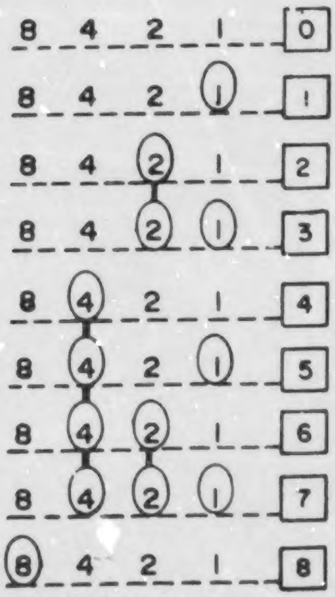

NAME
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1
8 4 2 1

More windows and panes.

Rule: If you place one counter on any "pane," you must place one counter on each pane in that "window" . . . only one counter on each pane.

Records of combinations are indicated by "looping" or otherwise indicating the window's use.

A nice pattern emerges:


8 4 2 1 0
8 4 2 1 1
8 4 2 1 2
8 4 2 1 3
8 4 2 1 4
8 4 2 1 5
8 4 2 1 6
8 4 2 1 7
8 4 2 1 8

Notice the columns of "1's" . . . one looped, one not looped . . . every other "1" is looped.

In the column of "2's," once it is started, two are looped, the next two are not looped, etc.

In the column of "4's," once it is underway, four are looped, and the next four are not.

Lesson II

I. Numbers and Operations.

   A.) Base and Place Value.

      1.) Using multi-base block as a model, develop ideas of base and place value. Use some base other than ten, then discuss these concepts.

      2.) Explore chip-trading games. Discuss the relationship of this activity to the preceding one.

      3.) Work through appropriate pages from Drill and Practice, a Problem-Solving Approach.

   B.) Statistics.

      1.) Organize, assign, and make graphs from data collected for Lesson I assignment. Discuss.

   C.) Geometry and Measurement.

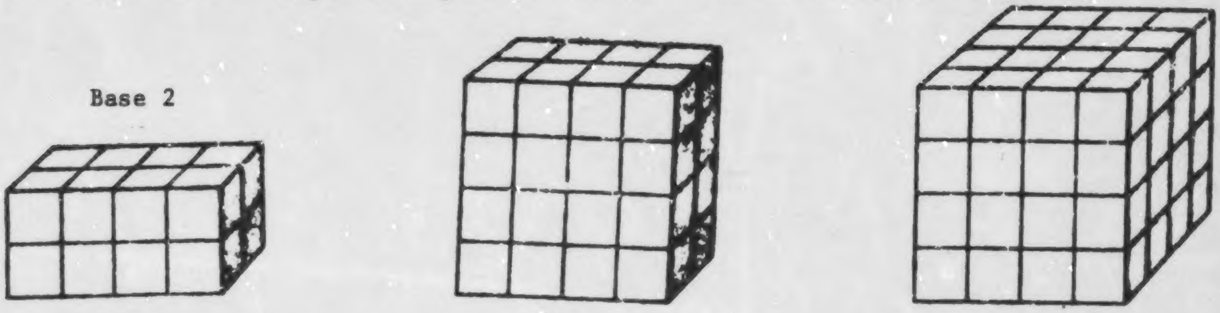
      1.) Using tiles, make rectangles of various sizes, keeping a record of area, perimeter, length and width.

      2.) Develop idea that area may remain constant, while perimeter changes, that perimeter may remain constant with a changing area.

Work in groups of four according to the following procedure. (Do not use base 10 wood for this game.)

A. Divide each group into two teams.

B. Place the wood representing one of the bases in the center of the table.



Long block
Fat long
Block long

Flat block
Fat flat
Block flat

Block block
Fat block
Block block

Figure 1.3

C. Have each team, in turn, roll the die. (Be sure to use the appropriate homemade die.) The number appearing on the first throw for each team will determine the number of blocks to be taken from the pile. The second throw will determine the number of flats, the third throw the number of longs, and the fourth the number of units.

D. After each team has four throws, the teams will determine a winner by deciding which team has the most (volume) wood.

E. Repeat the game several times, until each team has won at least once.

F. Questions concerning the game:

1. Given that if, on the first throw, the teams roll different numbers, can you tell which team will win the game? Why?________________________
____

2. Given that if you were using wood and a die representing different bases, how would you answer Question 1? Why?________________________
____

3. Suppose you used base 3 wood with a conventional die. Would your answer to Question 1 be the same? Play the game this way.____________
____

Factory Game

Lesson III

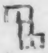
I. Logic

- A.) Sorting.
  - 1.) Sort people "things," such as rocks, postcards, etc. using ideas of intersecting sets and of disjoint sets.
  - 2.) Discuss rationale for this activity.

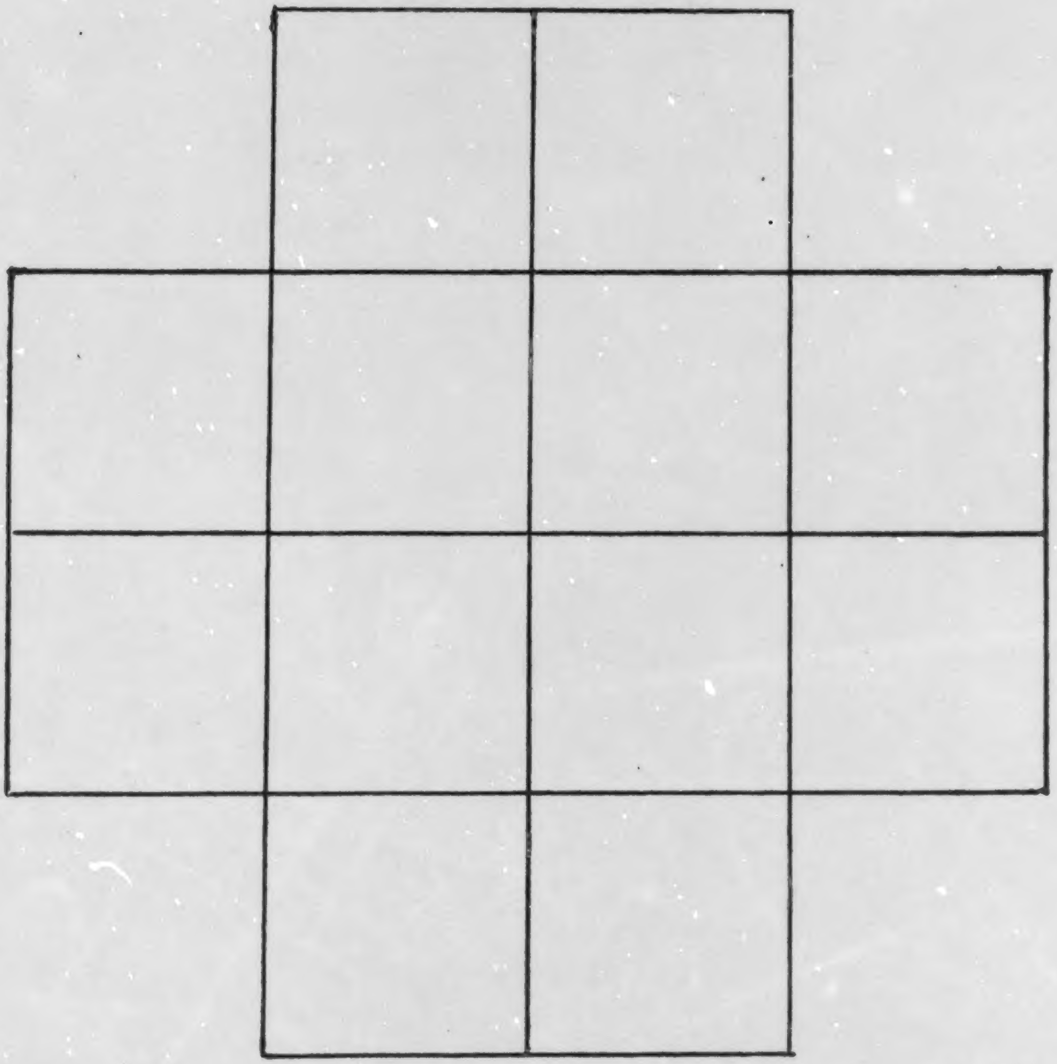
II. Measurement

- A.) Non-standard measurement.
  - 1.) Using strings, sticks, Cuisenaire rods, tiles, etc. as instruments, measure several objects. If different-sized instruments are used to measure the same object, a discussion should ensue concerning the necessity of standards.
- B.) The Metric System.
  - 1.) Make meter tapes, using the decimeter as the graduation. The orange Cuisenaire rod and construction paper can be used for this activity.
  - 2.) Find several objects about as long as the tape (meter), several about as long as one graduation (decimeter).
  - 3.) Add adhesive plastic tape graduated in millimeters to one decimeter. (This tape can be purchased at several math materials companies.) Find several objects which are approximately 1mm in length.
  - 4.) Discuss history of metric system, organization in tens, and prefixes: centi-, deci-, milli-, and kilo-.

III. Geometry and Problem-Solving

- A.) Using tiles, find the twelve pentaminoes. Discuss symmetry. The  leads to an introduction of the concept of point symmetry. Also, the question of the number of possible pentaminoes presents a challenge of explaining the fact that there are exactly twelve.
- B.) Give several pentamino problems.

FIND THE RECTANGLES



BY COMBINING THE RECTANGLES WHICH MAKE UP THE FIGURE ABOVE, YOU WILL BE ABLE TO DISCOVER QUITE A NUMBER OF RECTANGLES HIDDEN IN IT. HOW MANY ARE THERE IN ALL?

Lesson IV

I. Logic and Sets

- A.) Develop sorting abilities, using activities with Attribute blocks.
  - 1.) One-difference trains.
  - 2.) Cards showing one, two, three differences between blocks.
  - 3.) Intersecting sets.

II. Measurement

- A.) Develop concepts of Metric weight, liquid and Capacity Standards.
  - 1.) Using pan balance, balance scales, metric weights, metric volume and capacity containers, metric trundle wheel and meter sticks, set up a lab with task cards which the students are to accomplish.
  - 2.) Discuss tasks - simplicity of metric system.

III. Numbers and Operations

- A.) Review base and place value from Lesson II, using Multibase blocks. Discuss idea of dividing by tens in order to logically develop place value to the right of the decimal place.
  - 1.) Use the thousand unit block as ONE, and find models for .1, .01, .001.
  - 2.) Perform arithmetic operations using these blocks as models.
  - 3.) Relate this to metric system. Assign problems relating to this topic.

The Metric System

The standard linear measurement is called the

METER.

The closest English measure is the yard. It is about 3 inches shorter than the meter.

The standard weight measure is called the

GRAM.

About 33 grams ≅ 1 ounce.

The standard measure for small volumes is the

LITER

1 liter ≅ 1.06 U.S. quarts

Certain prefixes are used in all cases:

milli - meaning 1/1000

centi - meaning 1/100

deci - meaning 1/10

deca - meaning 10

hecto - meaning 100

kilo - meaning 1000

millimeter means ______ meter

centimeter means ______ meter

decimeter means ______ meter

decameter means ______ meters

hectometer means ______ meters

kilometer means ______ meters

The kilometer is 1000 meters.

(The mile is 1,760 yards)

This means that a kilometer is about .6 mile.

The Metric System

One gram of water is one cubic centimeter in volume. It is also one milliliter in volume.

The important prefixes are:

milli - centi - kilo -

Our numeration system is based on the number 10.

We have 10 digests: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

The place of each numeral is important:

153 does not have the same meaning which 315 or 531 or 513 has,

because 153 means 1 hundred, 5 tens, 3 ones.

"Deci-" is from the Latin word meaning ten. Our numeration system is called a decimal system.

ten thou. thousands hundreds tens ones

5 8, 9 6 4 .

To the left of the ones' place, we have powers of tens.

hundred = 10 X 10 or 10^2

thousand = (10 X 10) X 10 or 10^3

ten thousand = $\left[(10 \times 10)\right] \times 10 \times 10$ or 10^4

What happens to the right of the ten's place?

We're doing the opposite, dividing by ten.

. tenths hundredths thousandths ten thous.

. 2 1 3

This is read 4,213 ten thousandths.

1/10 = 1 divided by 10. 1/100 = 1 divided by 1000

1/10 = .1 1/100 = .01

The Metric System

100 centimeters = 100 cm = 1.00 meter = 1.00 m

105 centimeters = 105 cm = 1.05 meters = 1.05 m

203 centimeters = 203 cm = 2.03 meters = 2.03 m

810 cm = ______ m ______ cm = 2.60 m
463 cm = ______ m ______ cm = 8 m
982 cm = ______ m ______ cm = 4.5 m
1 cm = ______ m ______ cm = .5 m
45 cm = ______ m ______ cm = .32 m

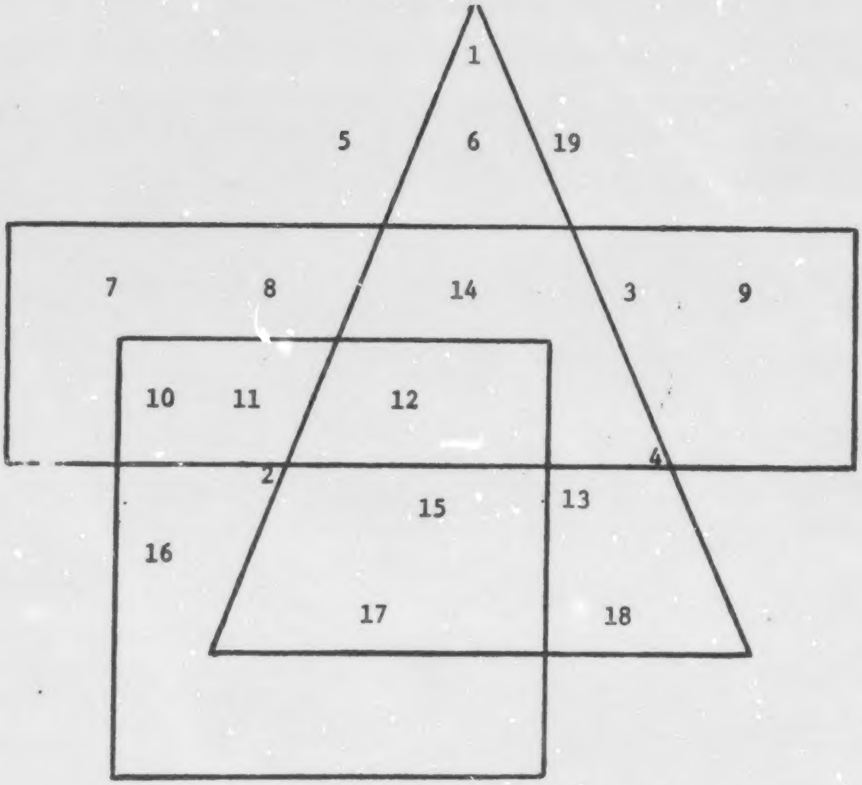
1000 milliliters = 1 liter

3000 ml = ______ liters ______ ml = 1 liter
420 ml = ______ liters ______ ml = 1.2 liters
6525 ml = ______ liters ______ ml = .001 liter

1000 milligrams = 1 gram 100 centigrams = 1 gram
250 milligrams = .25 grams 25 centigrams = .25 g
80 mg = .08 gr 85 centigrams = .85 g
4000 mg = ___ g 500 centigrams = ___ g

1000 grams = 1 kilogram
400 g = .4 kg
500 g = .5 kg
1500 g = __ kg

A NUMBER GAME



WHAT NUMBERS ARE:

1. In the rectangle, but not in the circle, square, or triangle?____________________

2. In the triangle, but not in the rectangle or square or circle?____________________

3. In the square, but not in the circle, triangle, or rectangle?____________________

4. In the circle, but not in the rectangle, triangle, or square?____________________

5. Common to both the rectangle and square?____________________

6. Common to both the rectangle and triangle?____________________

7. Common to both the rectangle and circle?____________________

8. Common to both the square and circle?____________________

9. Common to both the square and triangle?____________________

10. Common to both the triangle and circle?____________________

11. Common to all four figures?____________________

12. Now, add all of the numbers together; what is your total?____________________

Lesson V

I. Numbers and Operations

A.) Pre-Algebra.

1.) Presentation of ideas of true, false, open statements and the rule for substitution. This will be done using frames for variables rather than letters. Ideas for this presentation may be found in *Discovery in Mathematics*, Robert B. Davis, Addison-Wesley.

Examples:
$4 + 2 = 6$ True
$8 + 3 = 10$ False
$\square + 4 = 8$ Open

Rule for substitution: In an open sentence, the same number must be placed in all identical frames.

Example: In $\square + \square = 10$, 5 must be placed in each box. It is illegal to place 8 in one and 2 in the other.

2.) Function Machines and Formulas.

Ideas for this topic may be found in many standard text books, and in *Discovery in Mathematics*.

II. Measurement and Geometry

A.) Lab activities for the purpose of reinforcing ideas of metric measurement. Use same materials as in Lesson IV, with newly assigned tasks.

1.) Develop the concept via these tasks that 1 ml of water takes up 1 cc of volume and weighs 1 g.

B.) Using *Cubes, Rods and Squares*, develop idea of surface area and volume measurement, as related to solid shapes.

C.) Discuss ways of creating tasks for children for the purpose of reinforcing and expanding their understanding of these concepts.

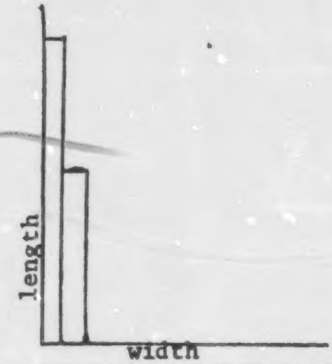
FUNCTION TASK CARDS

Rectangles

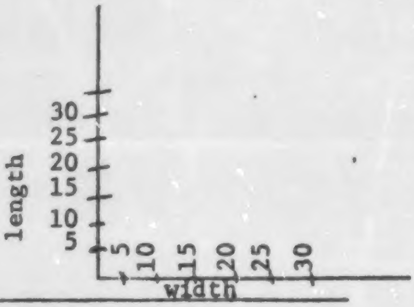
How many rectangles can you make whose area is 30 square units? Just use widths for which the length works out as a whole number. Make a table to record results:

Width of rectangle	1	2	3	5	6	etc.
Length of rectangle						

Draw a graph showing the actual rectangle. Use a big sheet of squared paper to draw on.

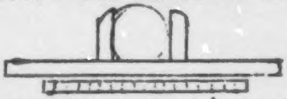


Plot a line graph of length and width.



CIRCLES

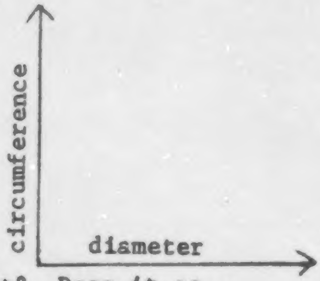
Collect several round cans or jars of different sizes. With the calipers and a ruler, measure the distance across the widest part of the round end.



With a tape measure or piece of string, measure around the cans also. (circumference)

Make a table like this:

Distance across can (diameter)			
Distance around can (circumference)			
Circumference Diameter			



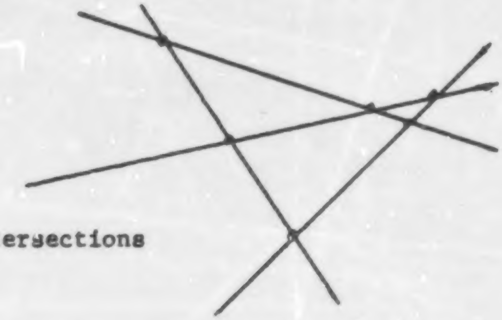
Draw a line graph of diameter and circumference. What shape is it? Does it go through origin? (o, 0) Why? What is the relationship between diameter and circumference? If you used big circles such as waste baskets, wheels, tires, etc., do you think you would find some relationship?

Lines and Intersections

Draw a number of lines and have them intersect the maximum number of times. Keep a record of the intersections and try to find the pattern.

Example:

4 lines



6 intersections

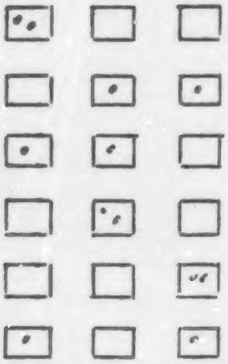
No. of lines	How many intersections
1	
2	
3	
4	6
5	

Beans and Boxes

How many ways can you put a given number of beans into 3 boxes? Keep a record and try to find the pattern.

Example:

2 beans:



Beans	No. of ways
0	
1	
2	6
3	
4	
5	

Lesson VI

I. Discuss various methods for multiplication. Additional examples can also be found in Drill and Practice at the Problem-Solving Level, Wirtz.

II. Relate all previous lessons to the texts being used in classrooms. Do this by topic - e.g. "Geometry - Symmetry."

III. Discuss any problems the paraprofessionals may have regarding topics or presentations in text.

Lesson VII

I. Review all topics previously investigated.

  A.) Numbers and Operations.

    1.) Base and Place Value. Questions and discussion to strengthen understandings and ability to use these concepts in computation.

    2.) Addition, Subtraction, Multiplication and Division Questions concerning the paraprofessional's ability to choose materials which will reinforce a student's understanding and skill in performing these basic operations.

    3.) Decimal Notation. Review the work accomplished in Lesson IV concerning this topic.

    4.) True, False, Open Statements. Review Lesson V.

  B.) Geometry.

    1) Review ideas of symmetry.

    2.) Discuss Topics in Lesson V.

    3.) Review geometric concept of similarity.

  C.) Measurement.

    1.) Review area, perimeter, volume and surface area functions, as presented in previous lessons.

  D.) Statistics.

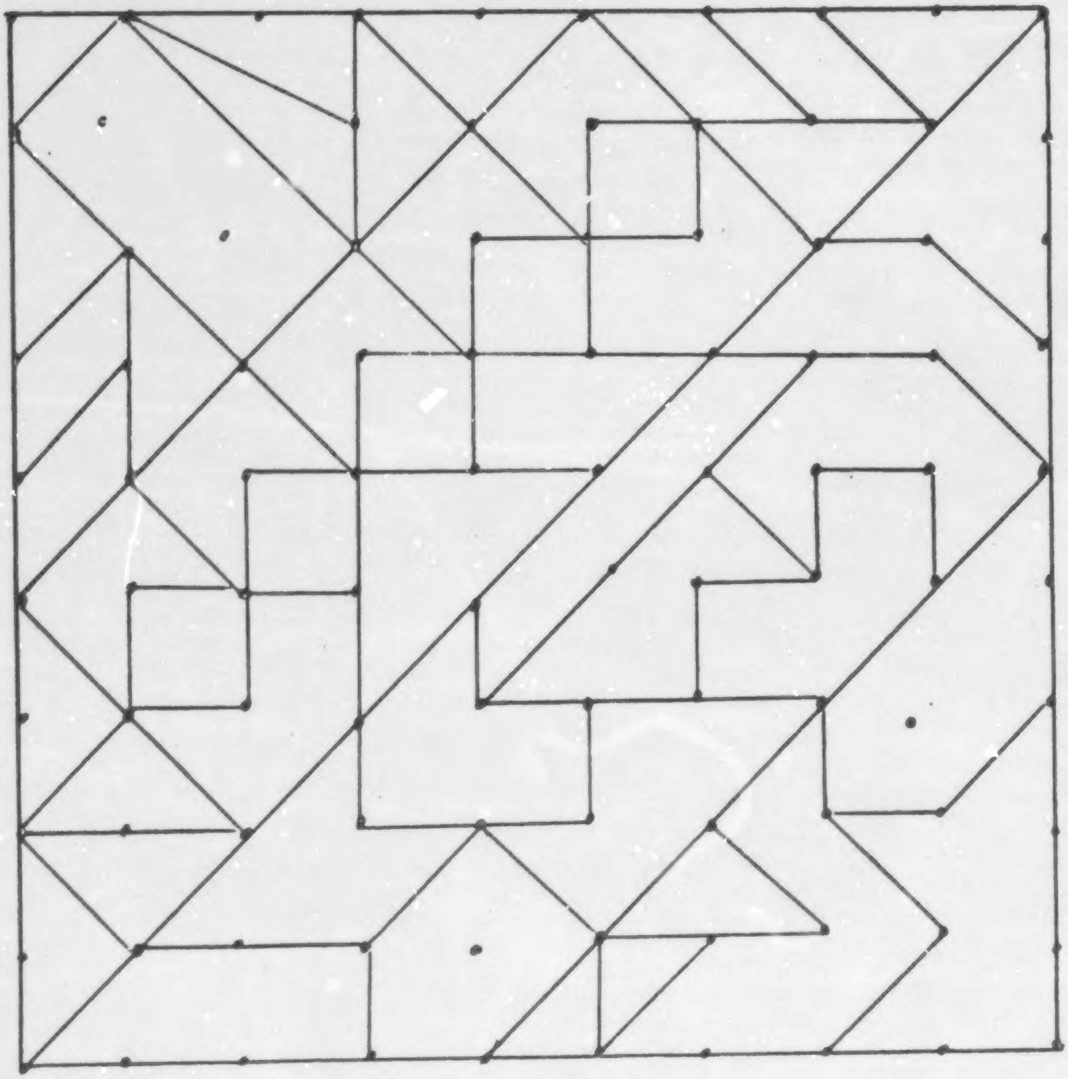
    1.) Discuss reasons for students to be able to read and interpret graphs.

    2.) Look at a graph. Interpret.

II. Give the paraprofessionals the opportunity to ask any questions they may have regarding any topics investigated.

GOOD EXERCISE FOR AREA

Color each area using the code below. When you have finished, can you figure out the total area that is blue?________________



1	- blue	2-1/2	- purple	3-1/2	- blue
1-1/2	- red	3	- purple	1/2	- blue
2	- red	4	- black		

= 1

MANY OF THESE GRAPHING EXERCISES CAN BE MADE FROM CHILDRENS' COLORING BOOKS.

BEGIN
10, 20
10, 22
5, 25
3, 27
3, 29
4, 31
3, 33
2, 36
1, 39
5, 39
9, 37
12, 38
15, 38
18, 37
22, 39
25, 39
25, 36
24, 31
24, 27
22, 24
19, 23
21, 21
23, 18
24, 16
24, 13
23, 11
22, 10
19, 9
15, 9
11, 10
10, 11
10, 12
11, 14
12, 15
14, 16
16, 16
17, 15
LIFT PENCIL
12, 21
10, 20
7, 18
6, 15
7, 14
7, 15
9, 17
10, 17
12, 17
14, 18
16, 20
LIFT PENCIL
10, 17
11, 14
LIFT PENCIL
24, 13
27, 12
28, 10
28, 8
27, 5
23, 3
19, 2
8, 2
5, 3
3, 3
1, 2
2, 6
3, 8
5, 9
7, 10
11, 10
LIFT PENCIL
4, 31
6, 32
9, 32
11, 31
12, 30
15, 32
18, 32
21, 31
24, 29
LIFT PENCIL
24, 27
22, 26
16, 26
12, 28
11, 27
9, 26
6, 26
3, 27
LIFT PENCIL
7, 28
6, 29
7, 30
8, 29
LIFT PENCIL
16, 29
17, 30
18, 29
17, 28
LIFT PENCIL
10, 25
12, 24
14, 25

LIFT PENCIL
11, 26
12, 27
13, 26
11, 26
LIFT PENCIL
24, 12
28, 8
LIFT PENCIL
23, 11
27, 5
LIFT PENCIL
22, 10
23, 3
LIFT PENCIL
18, 9
19, 2
LIFT PENCIL
15, 9
16, 2
LIFT PENCIL
11, 10
13, 2
LIFT PENCIL
8, 10
10, 2

LIFT PENCIL
5, 9
8, 2
LIFT PENCIL
3, 8
5, 3
LIFT PENCIL
2, 6
3, 3

Lesson VIII

I. Examination.

This examination should cover both the mathematics topics studied, and the ability of the paraprofessional to make judgments as to materials useful for reinforcing concepts for students and for remedial lessons.

TEST

1.) Write the answers:

a.) + 2 + - 3 = ___

b.) - 5 + - 6 = ___

c.) + 3 - + 8 = ___

d.) - 3 - +10 = ___

e.) - 6 x - 2 = ___

f.) + 6 x + 2 = ___

g.) - 6 x + 2 = ___

h.) + 6 x - 2 = ___

2.) If the statement is an identity, indicate by writing "I". If not, prove that it is not.

a.) □ + △ = △ + □ ___

b.) □ x 0 = 0 = ___

c.) □ x 3 = △ + △ + △ ___

d.) $□ \times \frac{1}{□} = 1 \quad (□ \neq 0)$ ___

Test (Cont.)

3.) The volume of the white cube is 1. What is the volume of each of the other nine cubes?

1.)

2.)

3.)

4.)

5.)

6.)

7.)

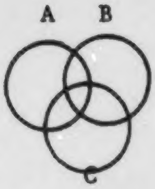
8.)

9.)

4.) Use the dark green rod as 1. Draw a picture showing 1/2 + 1/3.

5.) Use the brown rod as 1. Draw a picture showing 1/2 x 1/2.

Test (Cont.)



6.) Place three strings of A-Blocks:

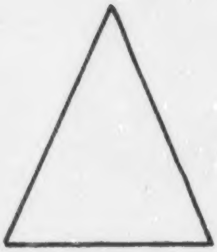
Place the small blue circle within all three strings: Describe subsets A, B, C so that the small blue circle belongs there.

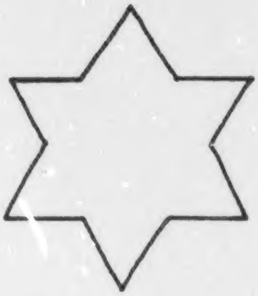
A:

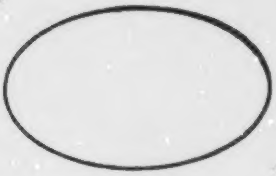
B:

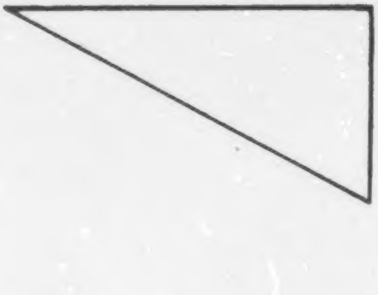
C:

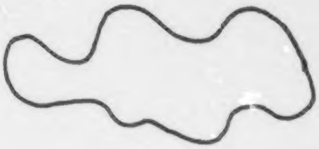
7.) If the figure has mirror symmetry, draw dotted lines to show this:







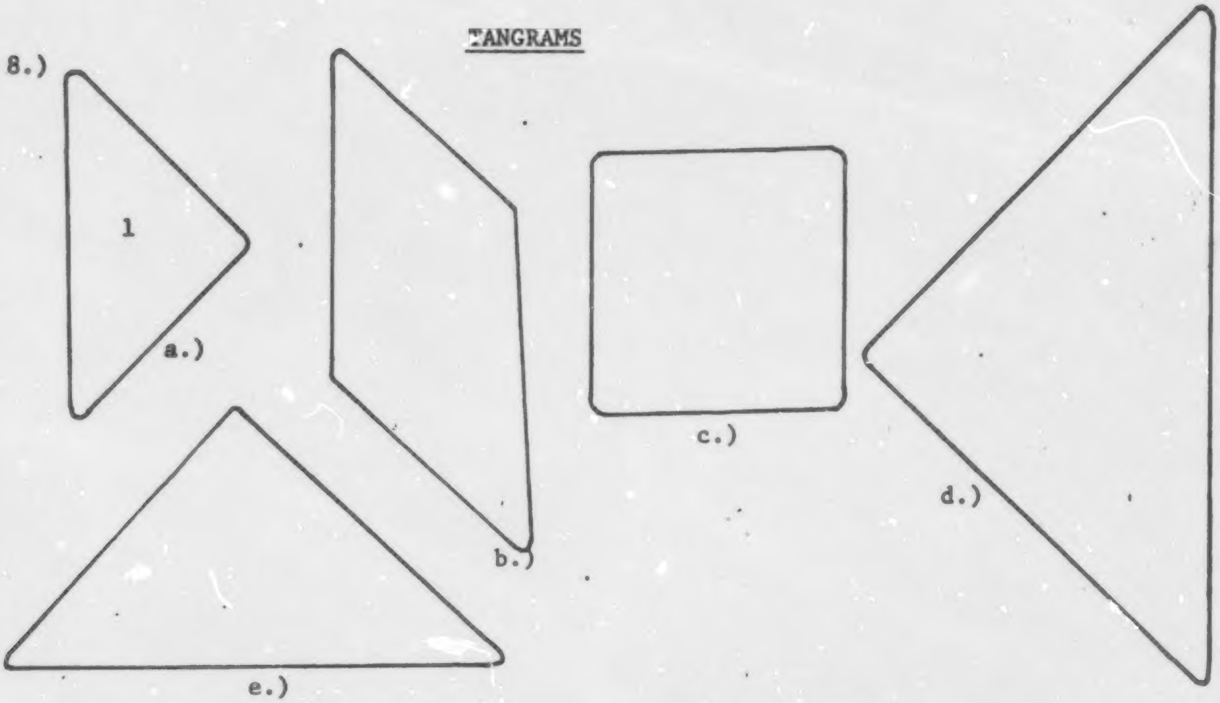




Test (Cont.)

TANGRAMS

8.)



The area of a.) is 1. Write the area of each other tangram piece.

9.) Can you make a square using a given number of tangram pieces? If so, write the letter name as in 8.) of pieces used.

1.) piece _____

2.) pieces _____

3.) pieces _____

4.) pieces _____

5.) pieces _____

6.) pieces _____

7.) pieces _____

Test (Cont.)

10.) With any materials available, construct an angle of:

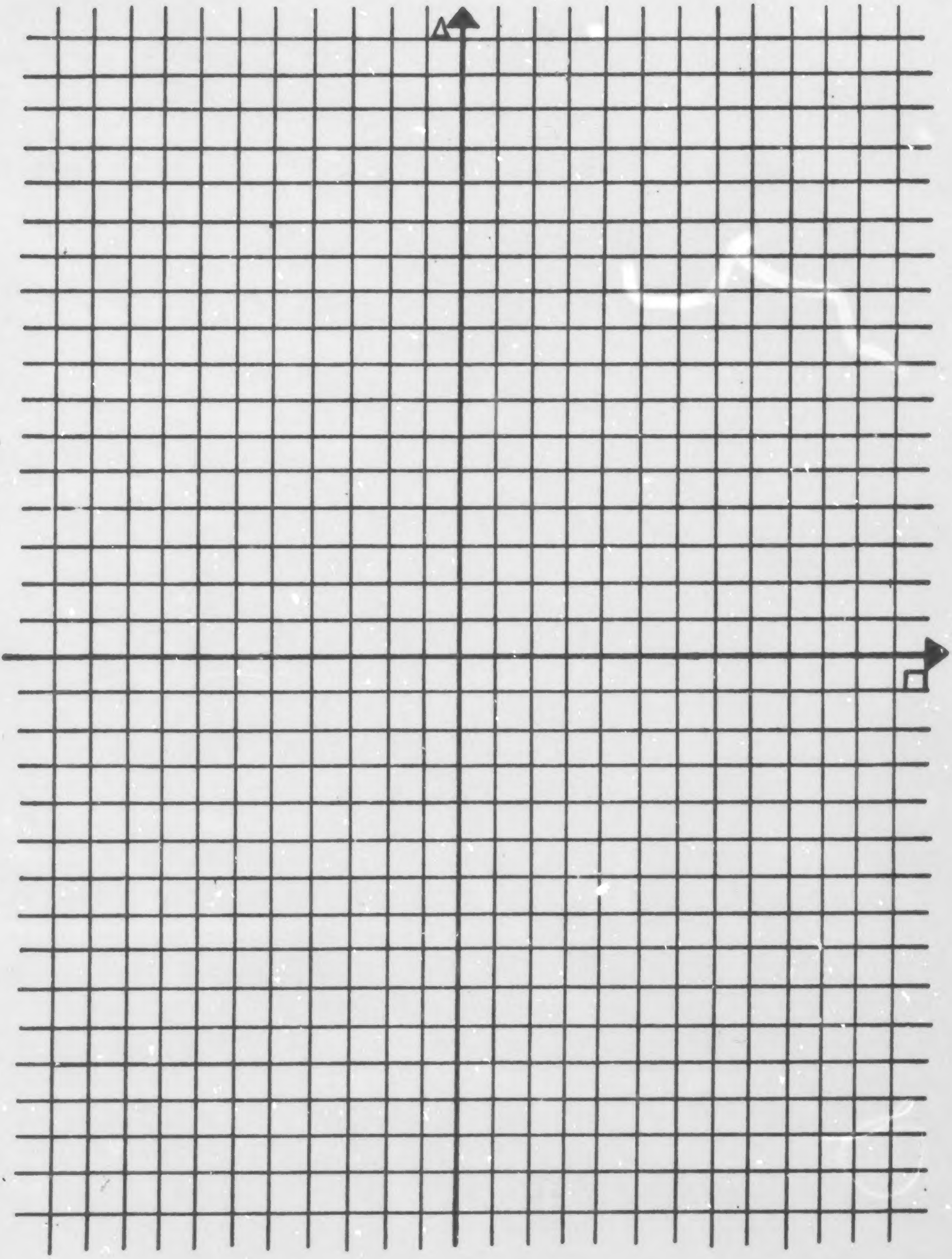
a.) 60° measurement

b.) 120° measurement

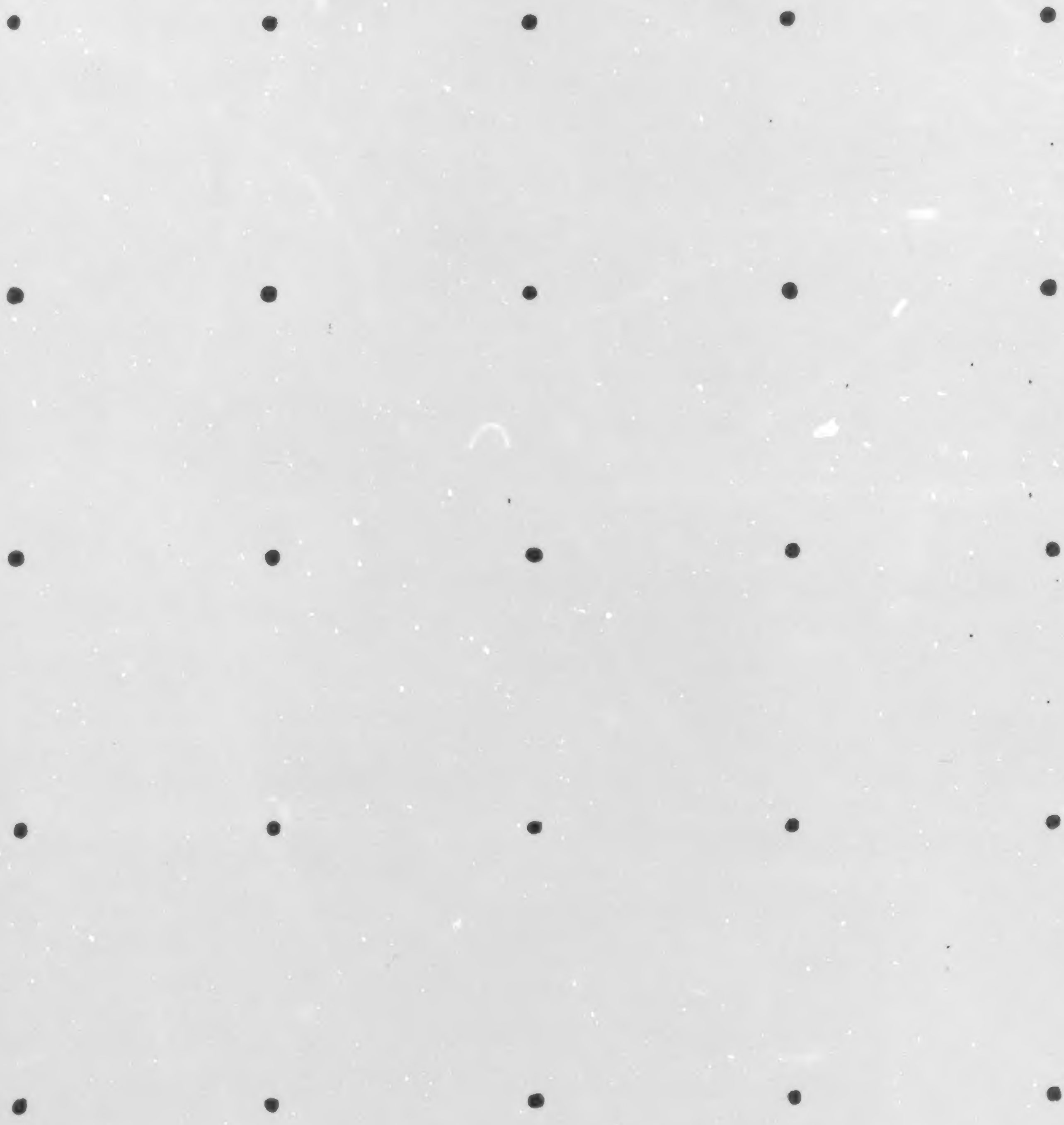
11.) Construct a rectangle on the geoboard which has an area of 3 units and a perimeter of 8 units. Draw this.

12.) Look at the pattern of blocks as I set them up, find the formula and graph.

□	△



Rule: ____________________



Lesson IX

I. Discuss examination given in previous lesson.

II. Measurement.

  A.) Set up a measurement laboratory in order to give the paraprofessional more experience in measurement tasks, and in using the metric system. This can be accomplished by having the proper materials and task cards.

  B.) Discuss the results of the experiments accomplished.

  C.) Have the paraprofessional write tasks which the students can perform in order to reinforce measurement concepts.

Lesson X

I. Pre-Algebra.

A.) Introduce positive and negative numbers, using several models. Ideas for this topic may be found in Discovery in Mathematics, Robert B. Davis.

B.) Review concepts of True, False, Open Statements, the rule for substitution and functions. Positive and negative numbers can now be used in the functions.

II. Functions, Measurement and Geometric Shapes.

A.) Use tiles, Geoblocks, Cuisenaire Rods, Cubes and Squares, Geoboards, Pattern Blocks to develop surface area functions and formulas of polygons. Deriving the function and formula from patterns found in empirical data is both enlightening and interesting for most students.

Time Trials

Use a watch with a minute hand to find out how long it takes you to count a hundred of anything. Example: Unifix cubes, bottles, caps, shells, beads. Be quick as you can. Now your partner will do the same. Now put away the watch and shells. Next, work out how long it would take you to count 500 shells. Write your answer in your book, telling how you did it. Now find out how long you would take to count 700 shells.

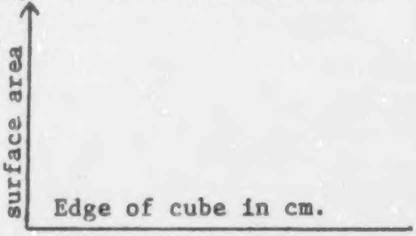
Cubes and Edges

You will need 64 one-centimeter cubes. Look at a one-centimeter cube. What is its surface area? Next, make a 2-centimeter cube. How many one-centimeter cubes do you need? What is the surface area? Repeat for 3-cm cube and 4-cm cube.

Make a table like this:

Edge of cube in cm	1	2	3	4	5	6	
Surface area of cube in cm							

Make a graph of these 2 measurements. Set it up like a graph.



What is the shape of the graph?

Does it go through the origin? (0,0)

What happens to surface area of a cube when you double the length of the edge?

What is relation between surface area and edge of cube?

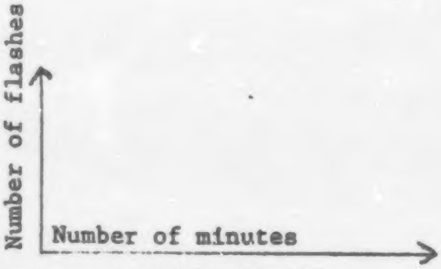
What is surface area of a metre cube?

Turn Indicator

You will need a stop watch, or watch with a second hand. You also will need the cooperation of an adult who will run his car for 5 minutes and put the turn indicator on. Start counting the flashes of the indicator and at the end of one minute call out the number of flashes; at the end of 2, 3, 4, and 5 minutes. Put your results on a table like this.

Number of minutes	1	2	3	4	5
Number of flashes					

Draw a line graph. →



What is shape of graph?
What is average number of flashes per minute?
If the indicator was left on for one hour, how many times would it flash?
Repeat this experiment with other cars.

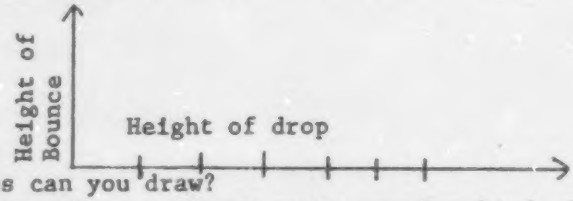
Bouncing Ball

Drop a ball from a height of 25cm. Have your partner note to what height the ball bounces, to the nearest cm. Do this 3 times from the same height to work out the average bounce.

Repeat this from 50 cm, 75 cm, 100 cm, and so on to 200cm. Work out a table like this:

Height of drop	Height of bounce (1) (2) (3)	Average height of bounce
25 cm 50 cm 75 cm		

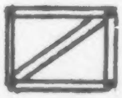
Draw a line graph of height of drop and average bounce. What shape is the graph? Does it start at origin? (0,0) Why?



What conclusions can you draw?
If you can, repeat the experiment using another kind of ball.

Making Things Rigid

You will need 8 cardboard strips 10 centimeters long. Also some spare strips to cut up as needed. Make a triangle from 3 of the strips. Use paper fasteners to hold the strips together. Can you change its shape by just pushing at the corners? It is a 'rigid shape'. Make a square from 4 of the strips. Can you change its shape by pushing at the corners?



Cut to size and fasten one extra to the square from one corner to the opposite one. Can you change its shape now? What new shapes have you made the squares into using one cross piece? How many of these new shapes are these?

Make shapes 5, 6, 7, and 8 sides and each time find the number of struts (cross pieces) needed to make the shape rigid always fastening them from one corner to the opposite corners.

Make a table of your findings:

Number of sides in shape						
Smallest number of struts needed to make rigid shape.						
Number of new shapes (triangles) formed						

Make a graph of:

1) Number of sides and number of struts needed.

2) Number of sides and number of triangles formed.

What is the relationship between:

a) number of sides and struts

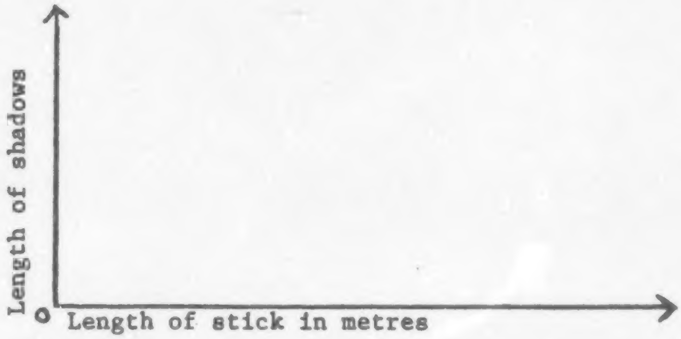
b) number of sides and number of triangles.

SHADOWS

You will need 8 sticks of length 10cm, 20cm, 30cm, 40cm, 50cm, 60cm, 70cm, 80cm. Hold each stick vertically on the ground while a partner measures the length of the shadow. How can you be sure that the stick is vertical? Why is it important? Record your results like this:

Length of stick (centimeters)	10cm	20cm	30cm	40cm	50cm	60cm	70cm	80cm	
Length of shadow (centimeters)									

Draw a graph of the results. What is the shape? Does it start at origin? (0,0) Why?



Find the length of shadow formed by sticks 1 metre and 2 metre long, from the graph. How could you check this number?

What is relationship between length of stick and length of shadow?

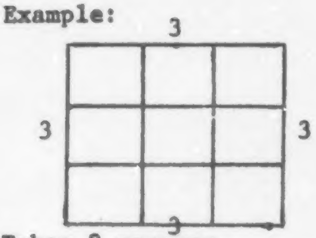
Can you find the height of some trees, telegraph poles, etc. by measuring their shadows?

What does the time of day you measure have to do with this experiment?

Squares from Squares (Pattern Blocks)

Using squares find out how many it takes to make bigger and bigger squares. Keep a record and try to find the pattern.

Example:



Takes 9 squares.

Length of side	No. of squares
1	
2	
3	9
4	
.	
.	
.	

Tower of Hanoi

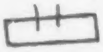
The temple in Hanoi has 100 golden circles which the priests move one at a time. When all are moved from one past to another world will end (it is said). Should we prepare? Move one at a time and a larger disc may never be put on top of a smaller.

How many moves would be necessary to transfer 1 disc from one past to another? 2 discs? 3 discs?

Can you write an equation?

# Discs	Moves
1	
2	
3	
4	
.	
.	
.	

Eight Men On A Raft



Start with one pair.
How many moves to change places?
Then use 2 pairs.
How many moves?
Do 3 and 4 pairs.
Can you predict how many moves it would take to have 10 pairs of men change places?
Can you write an equation for this function?
No fair jumping over 2 men at one time!!!!

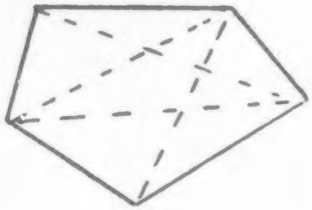
PAIRS	MOVES
1	
2	
3	
4	
.	
.	
.	
10	

Diagonals of polygons

Draw polygons with different number of sides. Draw diagonals in each polygons and record. Find a pattern.

Example:

Five-sided Polygon



5

diagonals

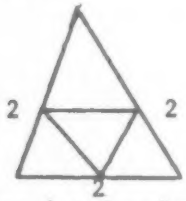
No. of sides	No. of diag.
3	
4	
5	5
6	
.	
.	
.	

Triangles from Triangles

Using triangles, find out how many it takes to make larger and larger triangles.

Keep a record and try to find a pattern.

Example:



Takes 4 triangles

Length of Side	No. of triangles
1	
2	4
3	
4	

Dienes Block Function

Materials: Dienes Blocks. One each from Base 3, 4, 5, 6, and 10.

If these blocks were painted red on the outside and then sawed apart, how many would have red paint?
on 3 sides?
on 2 sides?
...1 side?
...no sides?
Is there a pattern?

		0 sides	1 sides	2 sides	3 sides
3x3x3	1				
4x4x4	2				
5x5x5	3				
6x6x6	4				
10x10x10	5				

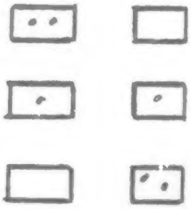
How Many Ways

How many ways can you put a given number of beans into 2 containers?

Keep a record and try to find a pattern.

Example:

2 Beans



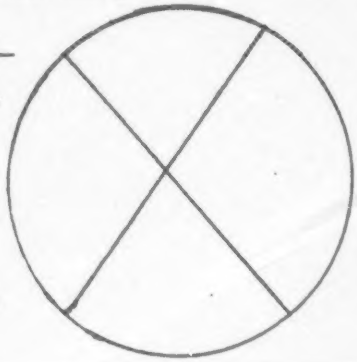
Beans	No. of ways
0	
1	
2	3
3	
4	
.	
.	
.	
.	

Cut the Pie

Cut a pie making the maximum number of pieces with each cut. Keep a record and try to find the pattern.

Example:

two cuts



4 pieces

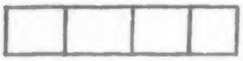
No. of cuts	No. of pieces
0	
1	
2	4
3	
4	
.	
.	
.	

Stamping Rods

Using a white Cuisenaire Rod as a stamp! Stamp the surface of other rods. Keep a record of the number of stamps on each rod. Try to find the pattern.

Example:

Purple Rod



Four stamps on each side = 16

1 stamp on each end = 2

Number of stamps 18

White Rod = 1	No. of stamps
(white) 1	
(red) 2	
(green) 3	
(purple) 4	18
. .	
. .	
. .	

Lesson XI

I. Numbers and Operations.

- A.) Discuss meanings of "fraction."
- B.) Using Cuisenaire rods, develop concept of equivalent fractions, and the necessity of understanding the unit to which each fraction refers.
- C.) Present development of fractions from Wirtz, Drill and Practice from a Problem-Solving Approach.

II. Functions.

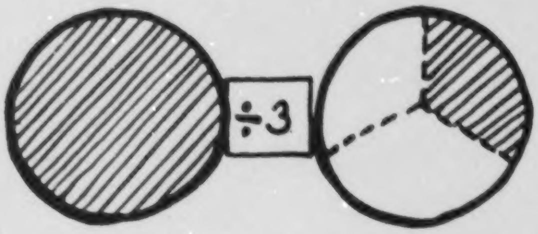
- A.) Outline a function task for each student which consists of:
  - 1.) Gathering data from a physical situation.
  - 2.) Organizing the data.
  - 3.) Graphing the number pairs.
  - 4.) Deriving the rule.
- B.) Discuss the outcomes.

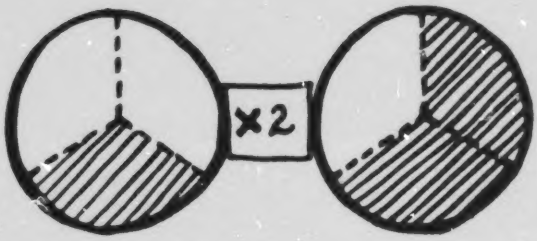
III. Measurement (Scaling).

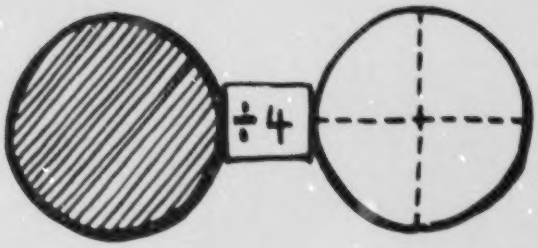
- A.) Measure (using metric standards) some area -- the classroom, basketball court, etc. The students may be assigned in pairs to measure a specific part of the area.
- B.) Discuss ways for making a blueprint of the measured space, and a scale. If possible, 1:10 would be a very helpful ratio for this scaling project.
- C.) Using graph paper, produce a blueprint. If the area is large, and has quite a bit of furniture, this task may be assigned in part to each pair of students, then put together as an end result.

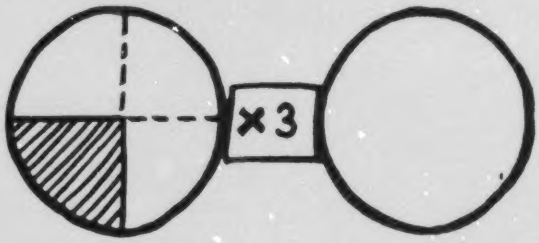
÷3　　×2

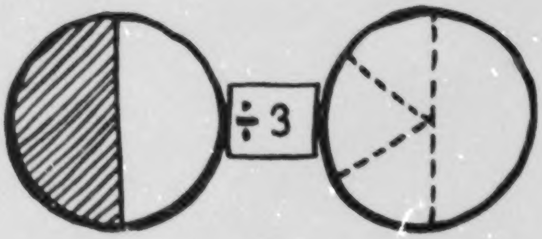
DIRECTIONS TO "DO SOMETHING"
DIRECCIONES PARA "HACER ALGO"

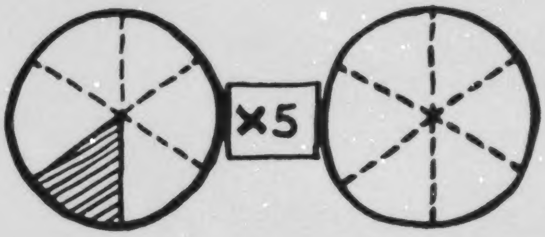


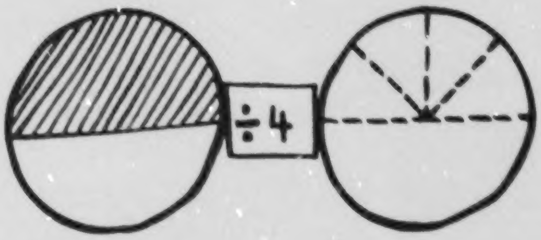


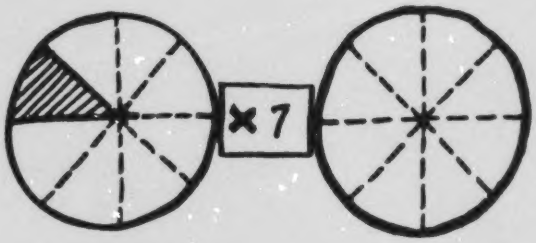


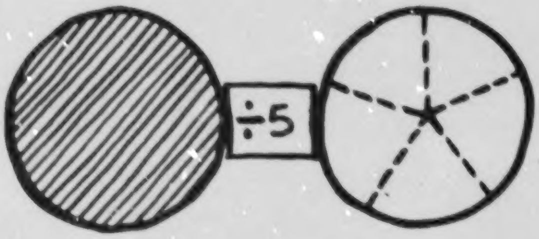


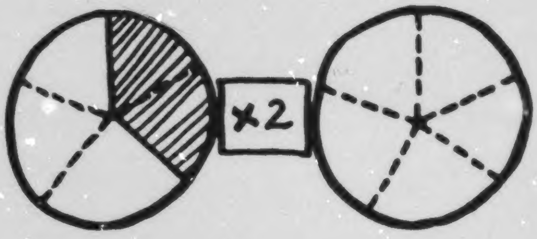












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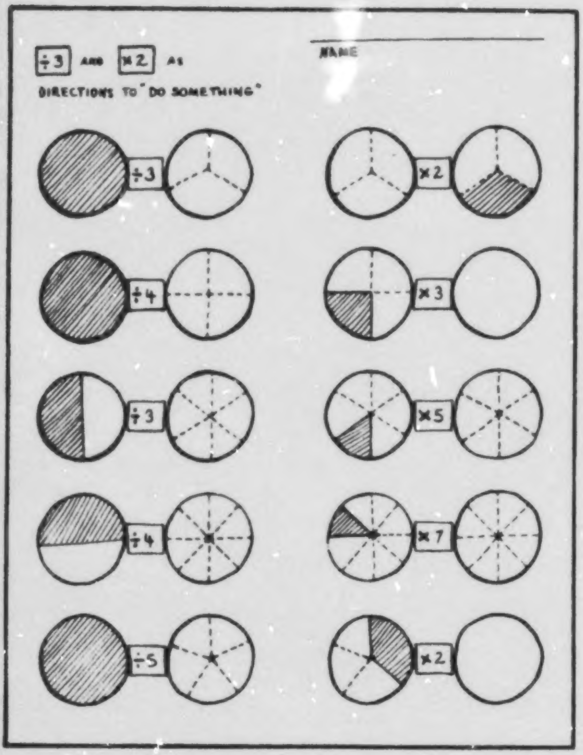
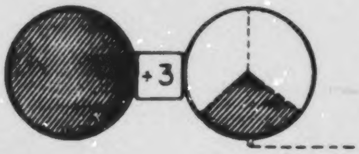


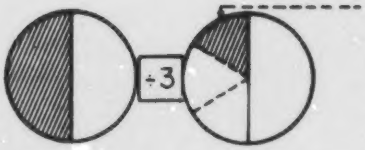
This page begins building toward understanding fractions and multiplication by fractions. Examples in the left hand column ask the learner to divide the shaded area into a certain number of equal pieces and to shade in one of those pieces.

The task can be made clearer for some learners by asking the question as follows: "The shaded area represents a pie (or pizza, or cookie) or some part of a pie. The number in the square tells you how many children are going to share the pie. Please cut up the pie so that each child will get the same amount, and shade one child's portion." Thus, in the first example the solution would be:



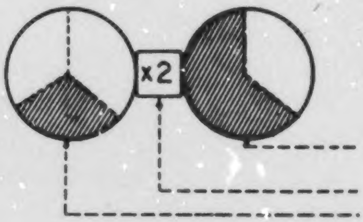
Each child gets this much.

The third and fourth examples on the left have a somewhat new twist. There is only half a pie to start with. But the question remains the same. Thus:



Each child gets this much.

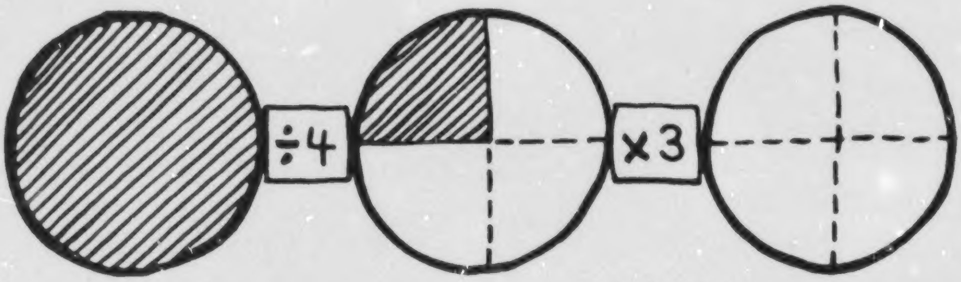
In the right hand column of examples, the circle on the left shows a portion shaded. The number in the box tells the learner how many such portions to shade in the right hand circle. In terms of "pies and people," the question now is: "The shaded part in the left hand circle shows the amount of pie to be given each person. The number in the box tells how many people are to be served. How much of the pie will be used up?" Shade that amount in the circle on the right. Thus:

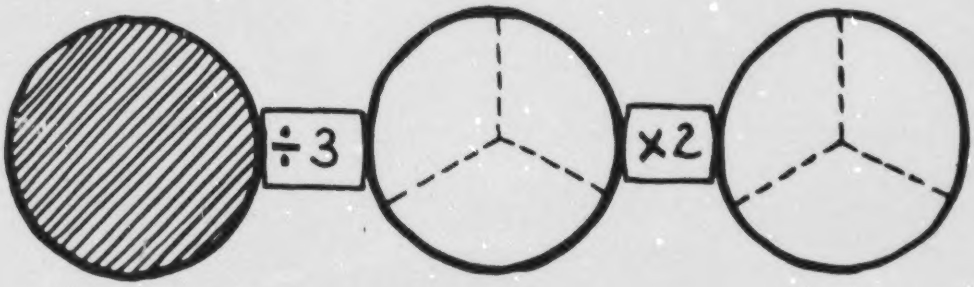


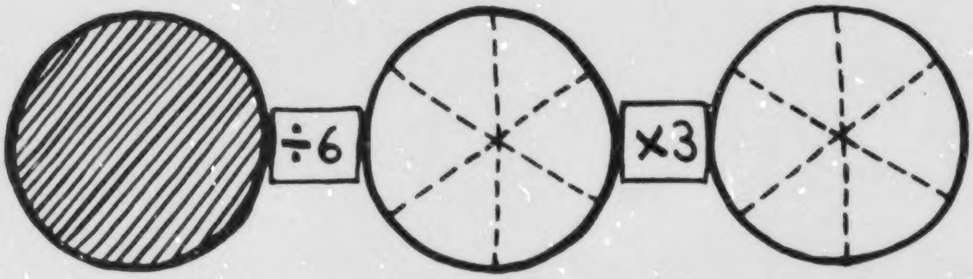
This amount of pie will be used if two people get servings this big.

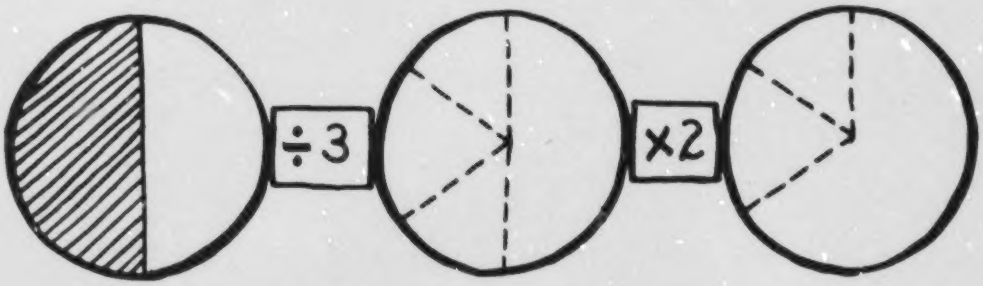
÷ a   × b

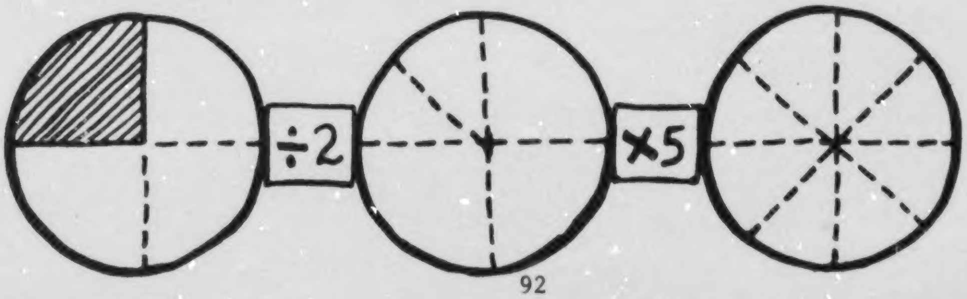
____











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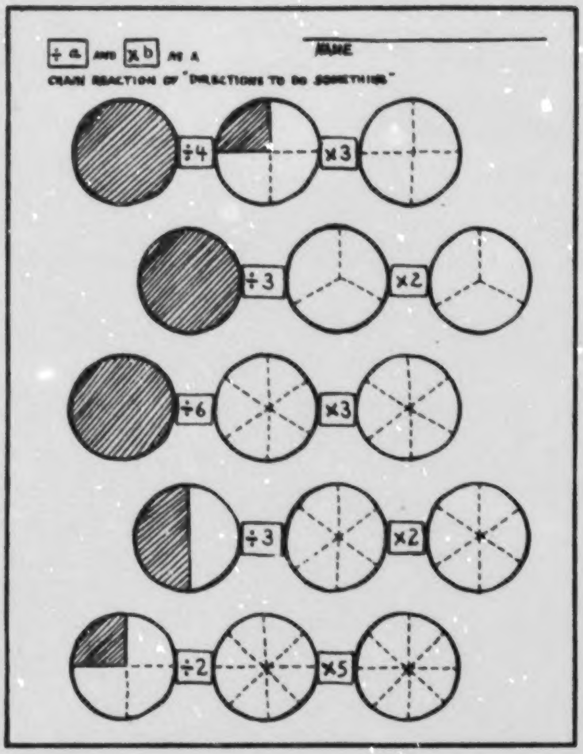
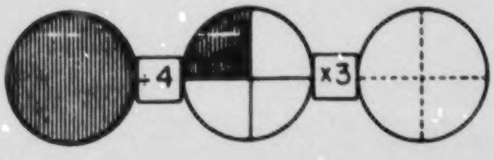
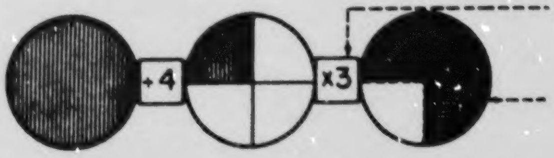


This page compresses the activities on the previous page, so that dividing the circle and "multiplying" a part of it are done all in one "chain reaction." In terms of our "pies and people" scenario, the story might be: "I start with the amount of pie shaded in the first circle. It is to be cut into the number of equal pieces indicated in the box. Please shade in one portion."



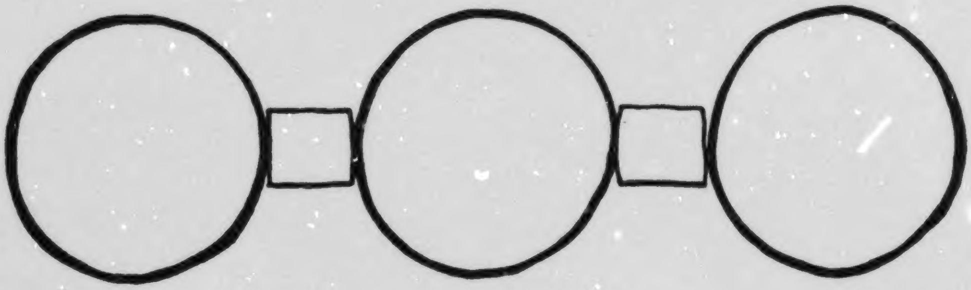
"Now, if I want to give that size portion to the number of people in the next box how much pie would I need?"

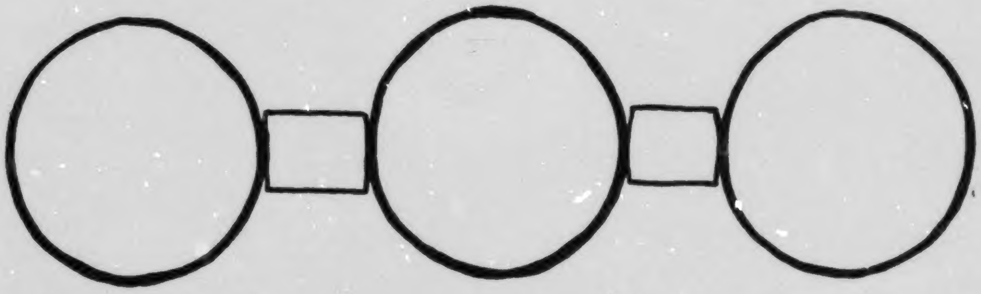


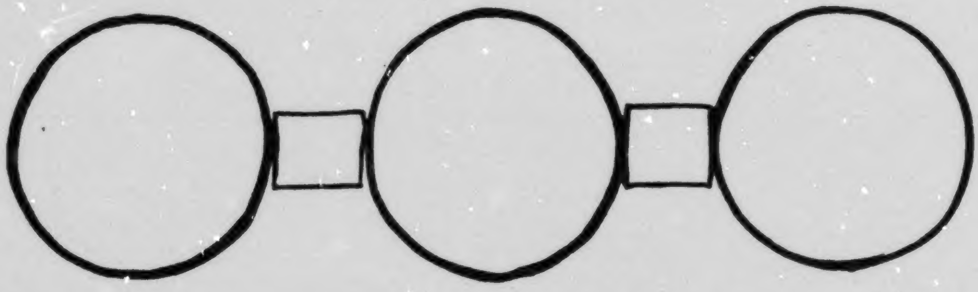
To serve 3 people I need this much pie.

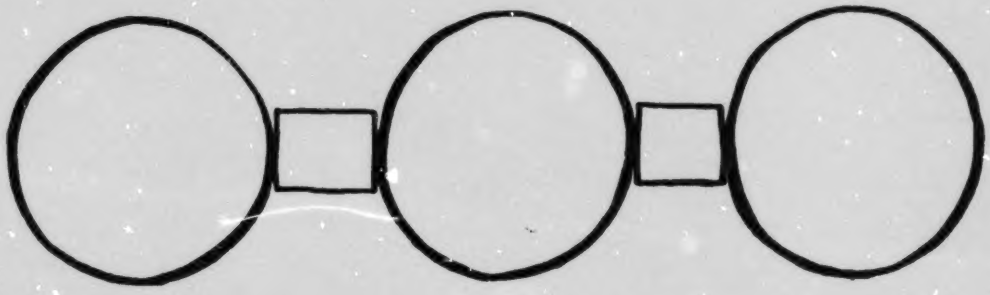
In the last example on this activity page, it turns out that we would need more pie than we started with. In this case, we could add to our story that we have some other pies on hand for when this situation arises.

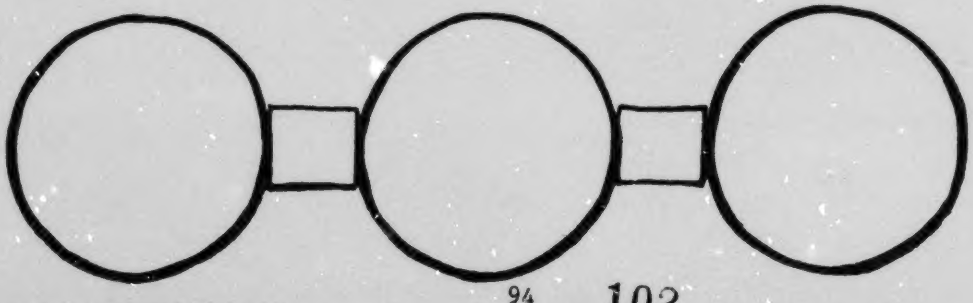
$\boxed{\div a}$ $\boxed{\times b}$











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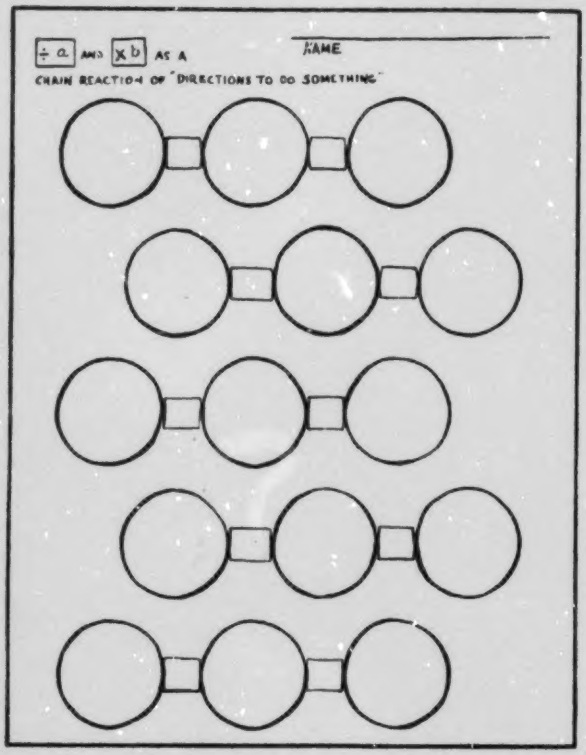


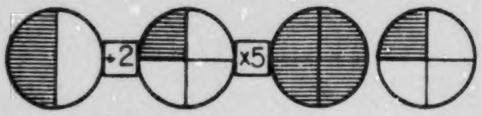
÷ a AND x b AS A

NAME

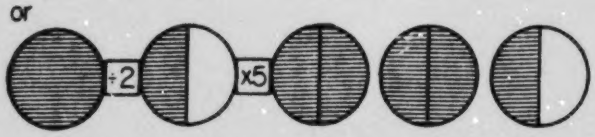
CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

This form enables learners to make up their own examples and carry out investigations. It can also be particularized by the teacher.

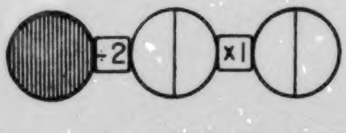
Some new situations might arise in which the learner will have to draw an additional circle or circles to complete the chain reaction. For example:

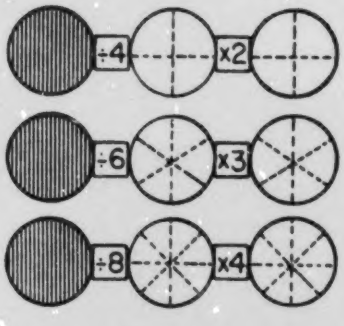


or

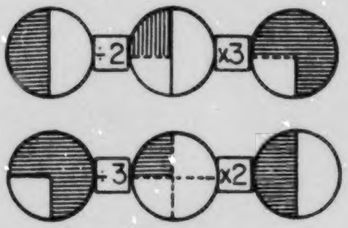


One possible way to particularize this page might be to structure some examples that build toward the idea that 1/2=2/4=3/6, etc. For example, sequences like the following could be designed:



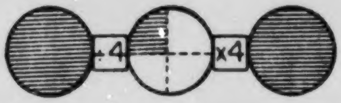


You might also want to structure examples which "undo" each other:



to build some referents for the later introduction of the notion a/b.b/a=1

Examples that do not change the original portion can be developed:

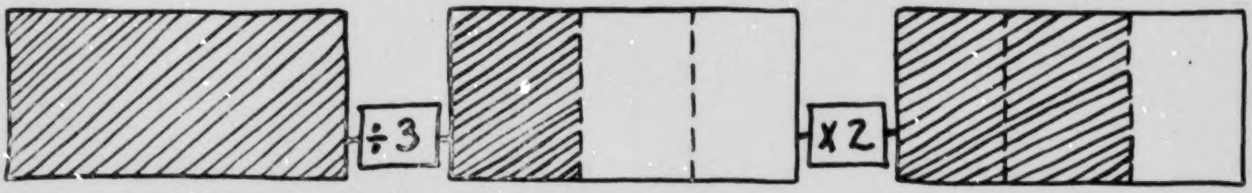


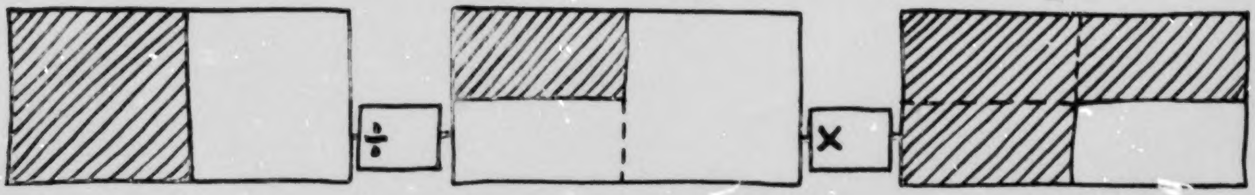
to build toward the later introduction of the idea that a/a=1.

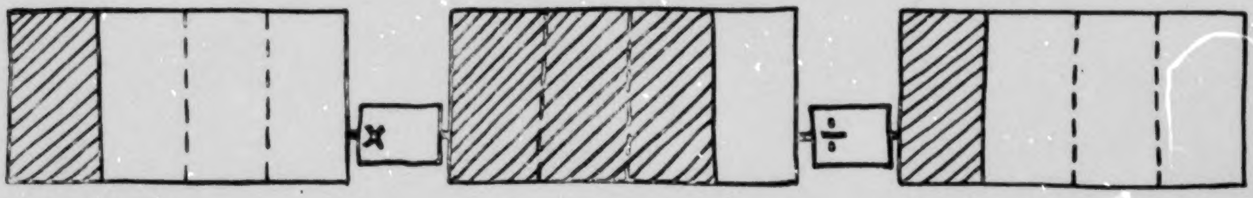
Once children see examples of this type, they could investigate to see whether such chain reactions have the same effect when different portions of the circle are used at the start.

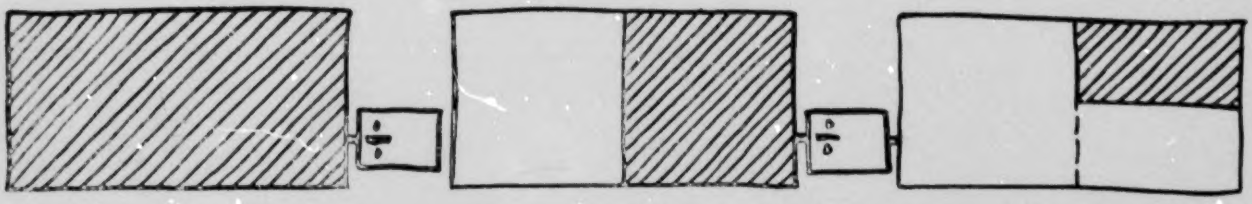
÷c ×d

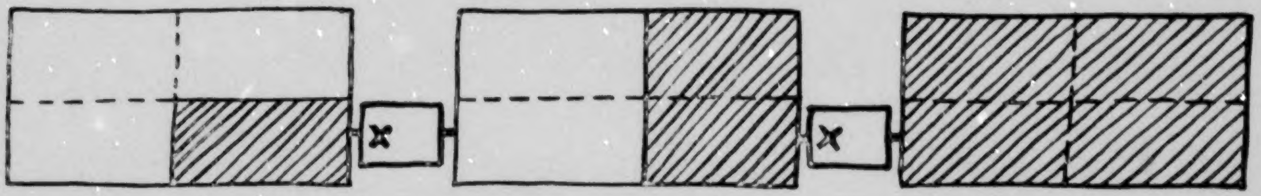
"DIRECTIONS TO DO SOMETHING"
DIRECCIONES PARA "HACER ALGO"

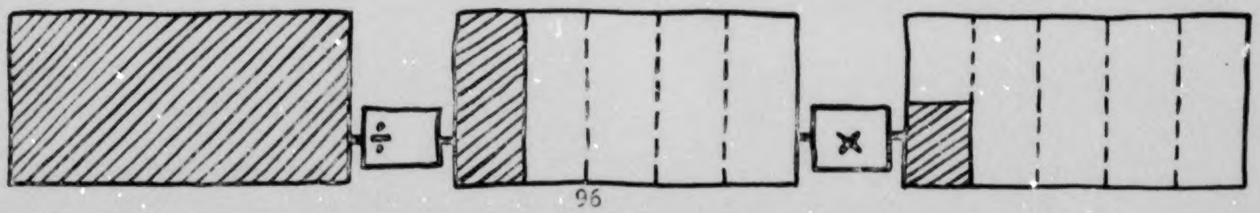












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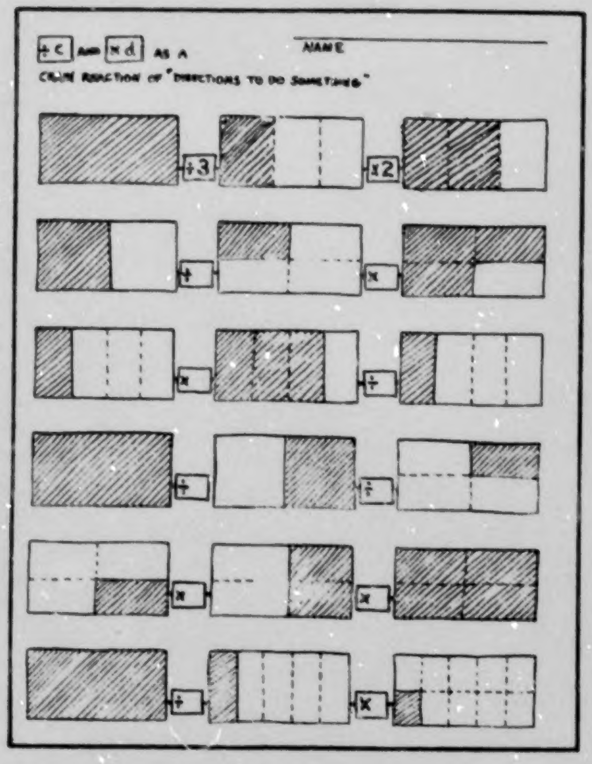
÷c AND ×d AS A
CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"
NAME
÷3
×2
÷
×
×
÷
÷
÷
×
×
÷
×

This activity is similar to those on the previous pages. But now a rectangle is used instead of a circle, and the results of the chain reaction are shown in the diagram. The learner must describe the chain reaction by writing in the appropriate numbers in the small boxes.

$\boxed{\div c}$ $\boxed{\times d}$

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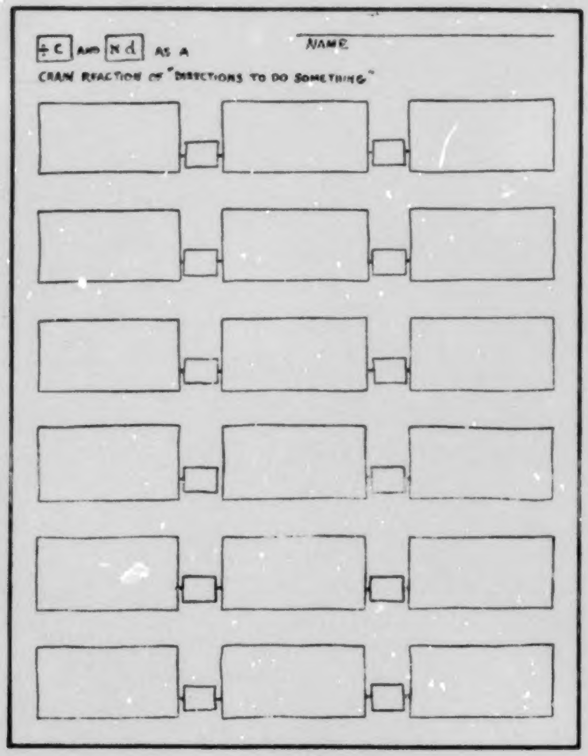


$\boxed{\div c}$ AND $\boxed{\times d}$ AS A

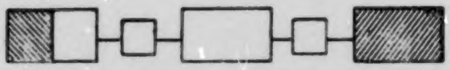
NAME

CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

This form can be used for a variety of investigations. Some possibilities are suggested on Page 78 C.

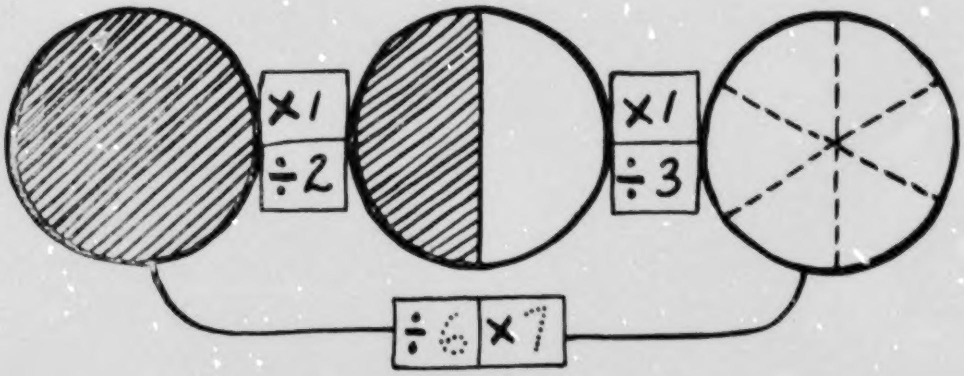
A question to explore:

Start with one half of the rectangle. Can you find a chain reaction that will result in doubling it? That is, what chain reaction could be written to fit:

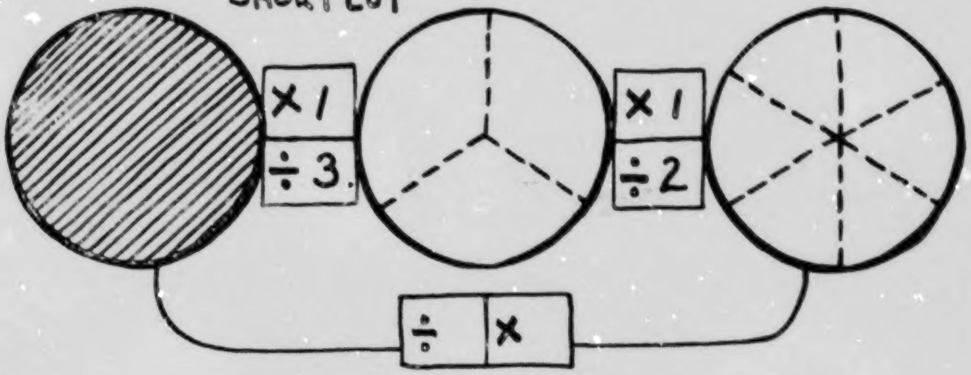


Is there more than one chain reaction that will work? Will these chain reactions "double" any area I start with?

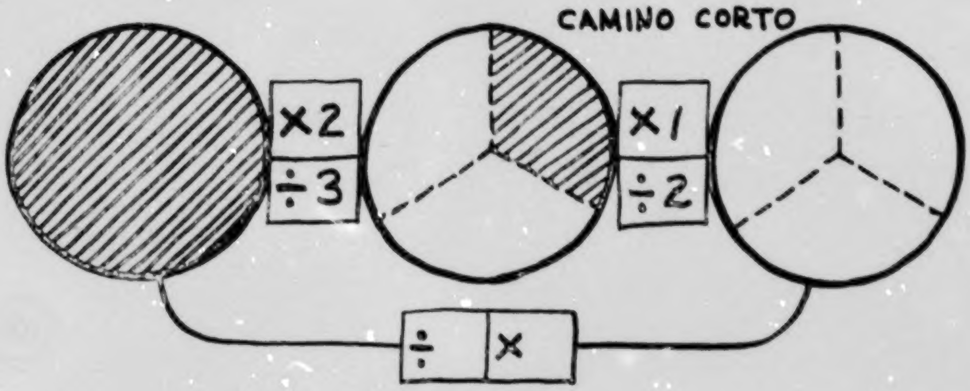
÷a ×b

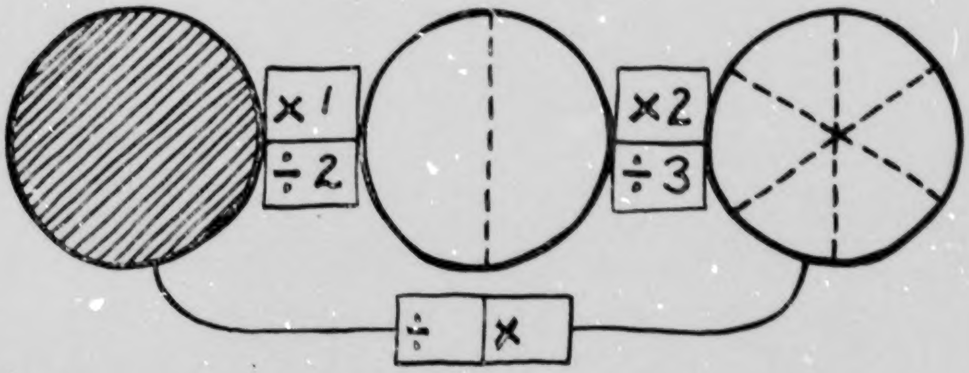


SHORTCUT



CAMINO CORTO





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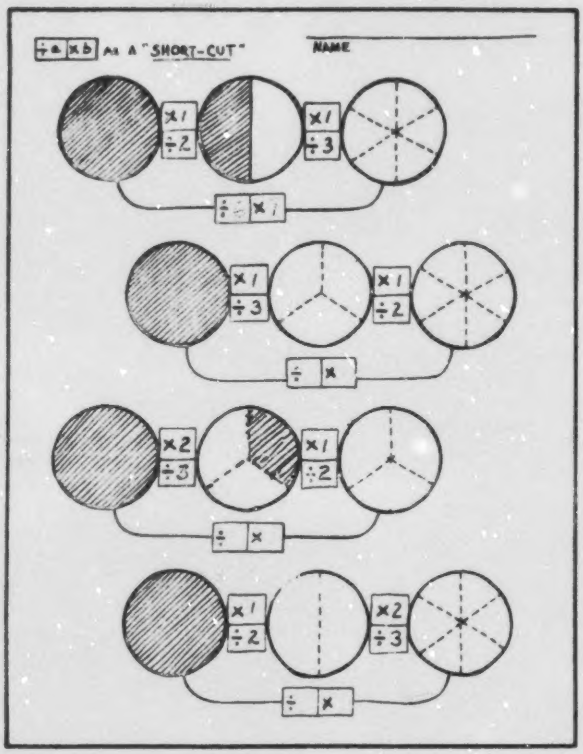


On this page there are two new twists. First, the chain reaction that previously was done in two separate steps must now be done in one step. Secondly, two chain reactions are done in sequence and the learner must find one chain reaction that would produce the same result. Referents are being built for fractions and multiplication by fractions.

If learners need a real situation which can give meaning to the "chain reaction" now expressed by

$$\boxed{\begin{array}{c}\times\ 1\\ \hline \div\ 2\end{array}}$$

it might be the following: "I start with the amount of pie shaded in the first circle. The box tells me

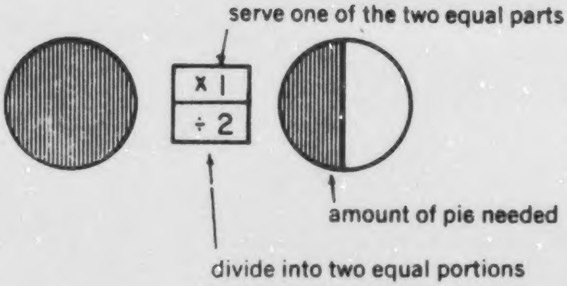
$$\boxed{\div\quad\quad}$$

how many equal parts I must divided the pie into. Each part is one portion of pie. The box tells

$$\boxed{\text{x}\quad\quad}$$

me how many portions I must serve. In the second circle, I should shade in the amount of pie I will need."

Thus:

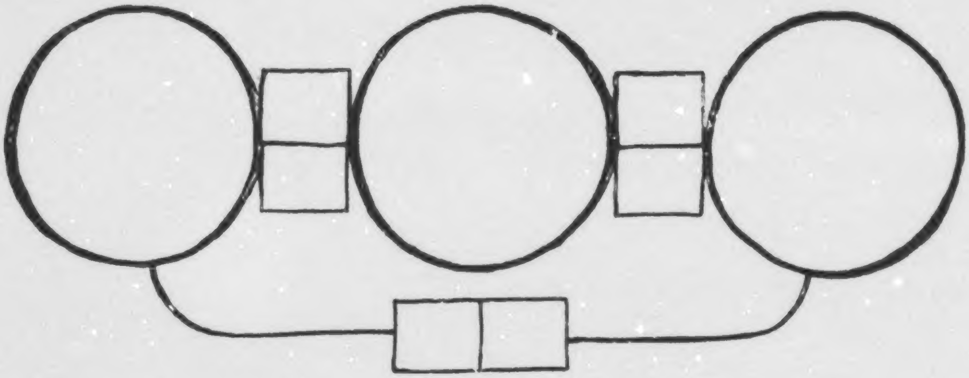

serve one of the two equal parts
x 1
÷ 2
amount of pie needed
divide into two equal portions

"The second chain reaction can be thought of in the same way, but now I will begin with a different amount of pie — that shown in the second circle."

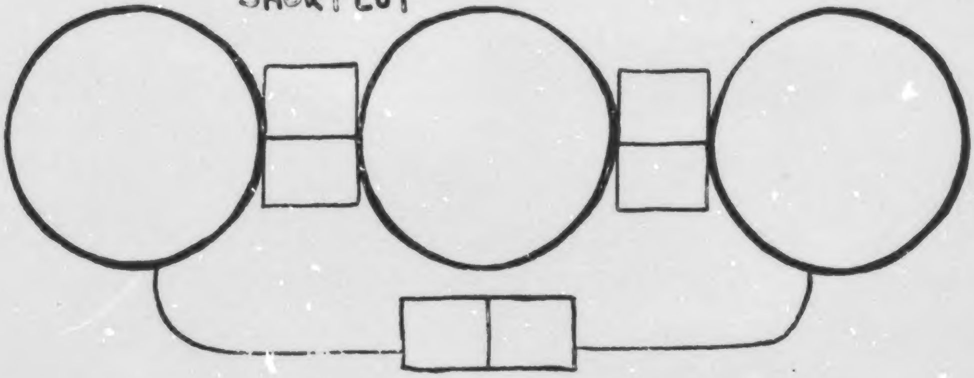
Once the result of the two chain reactions has been found, the problem becomes one of finding one chain reaction that would produce the same result.

On this page, the first two examples should be considered together, and the last two together. Each pair suggests an "in-any-order" rule for these chain reactions.

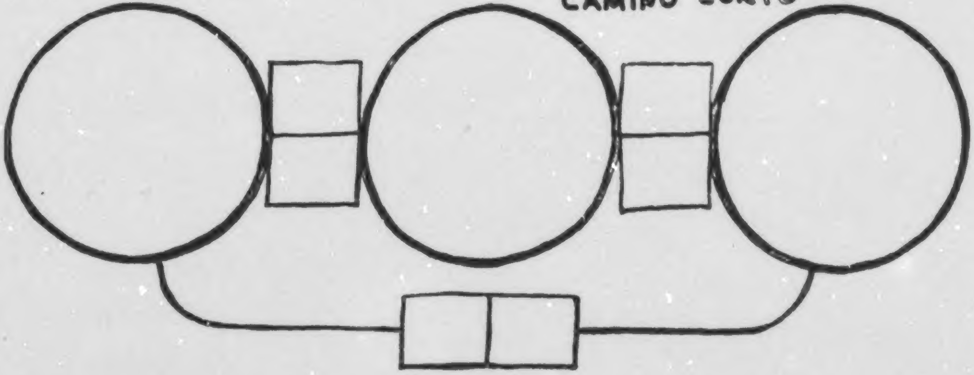
÷ a | × b

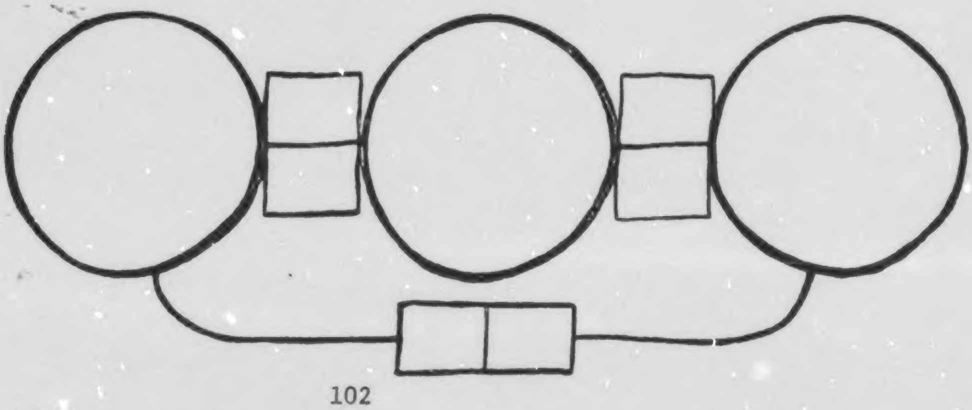


SHORTCUT



CAMINO CORTO





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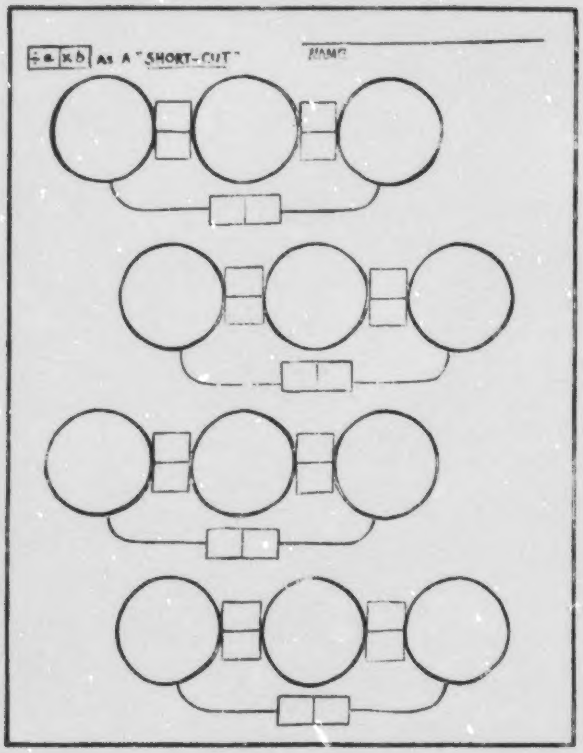
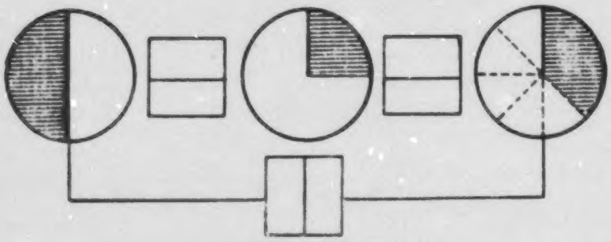


Learners can make up their own examples using this form, or it can be particularized by the teacher. A few questions to explore:

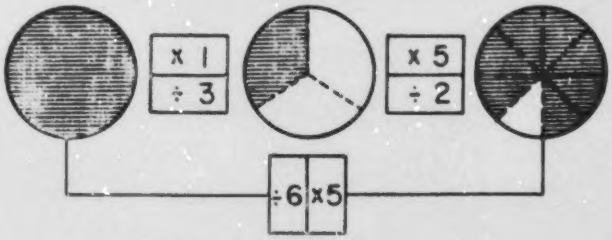
(1) Shade in parts of the first and third circles. Can you find chain reactions to fit? For example, what chain reactions could lead to this situation?



What shortcuts could be used?

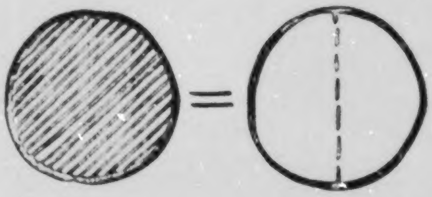
This might be a good opportunity to talk about shortcuts that use the smallest whole numbers, and agree that these shortcuts will be the ones we will record. In the example above, ÷2, ×1 was one possible shortcut. But we also might have used ÷4, ×2 or ÷8, ×4. But the shortcut ÷2, ×1 used the smallest whole numbers, so under the new agreement, that would be the shortcut we would record.

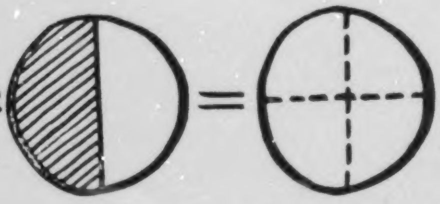
(2) If the only information given is the shortcut, can you find a "top line" to fit it? For example, if the shortcut were ÷6, ×5, one solution would be:

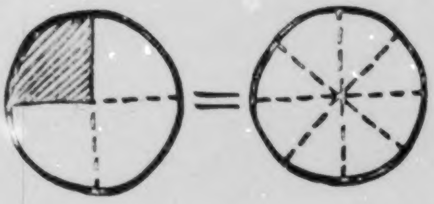


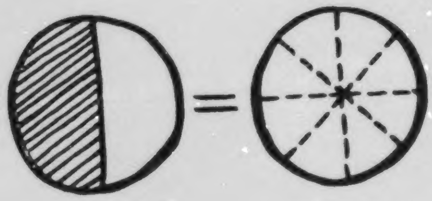
Can you find another?

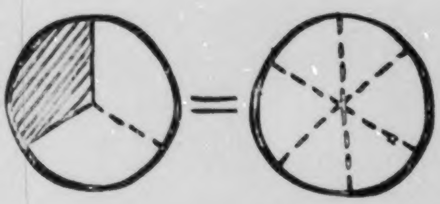
$\frac{1}{2}$ of 14 = 7 $\frac{1}{2} \times 14 = 7$

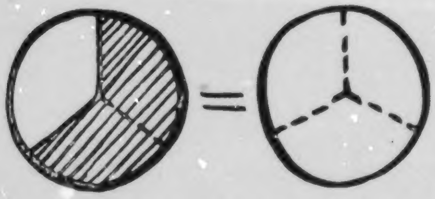
$\frac{1}{2}$ of


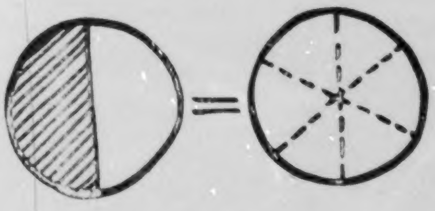
$\frac{1}{2}$ de


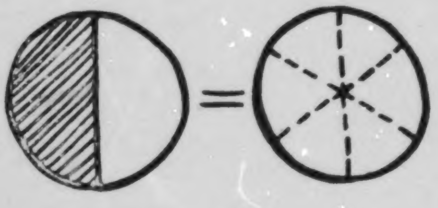
$\frac{1}{2}$ de


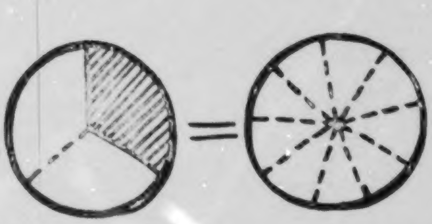
$\frac{1}{4}$ of


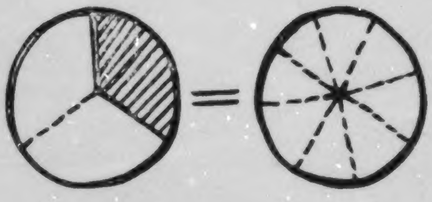
$\frac{1}{2}$ of


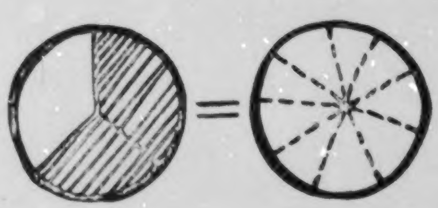
$\frac{1}{2}$ de


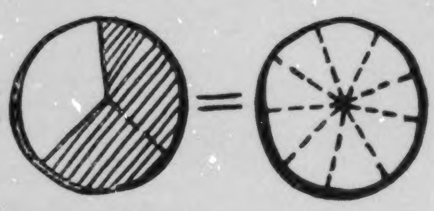
$\frac{1}{3}$ X


$\frac{2}{3}$ X


$\frac{1}{3}$ X


$\frac{2}{3}$ X


$\frac{2}{3}$ X


$\frac{1}{3}$ X


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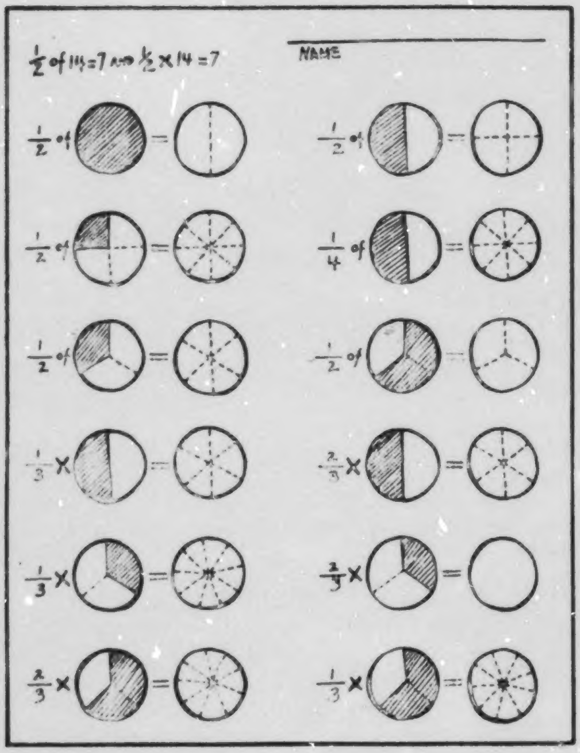
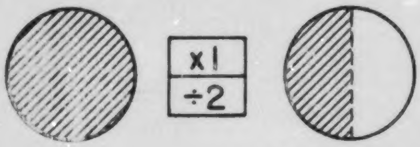
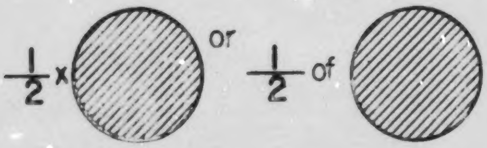

1/2 of 14 = 7 and 1/2 × 14 = 7
NAME
1/2 of =
1/2 of =
1/2 of =
1/4 of =
1/2 of =
1/2 of =
1/3 × =
2/3 × =
1/3 × =
2/3 × =
2/3 × =
1/3 × =

The "chain reaction" activities have grown up on this page and learners work with fractions in standard form. Instead of writing



we now write



The meaning is the same, however.

Children should be encourage to avoid doing two separate steps when they are certain of the outcome.

____

$\frac{1}{2}$ of 14 = 7 $\frac{1}{2}$ × 14 = 7

___ de ○ = ○

___ of ○ = ○

___ de ○ = ○

___ × ○ = ○

___ × ○ = ○

___ × ○ = ○

___ of ○ = ○

___ de ○ = ○

___ of ○ = ○

___ × ○ = ○

___ × ○ = ○

___ × ○ = ○

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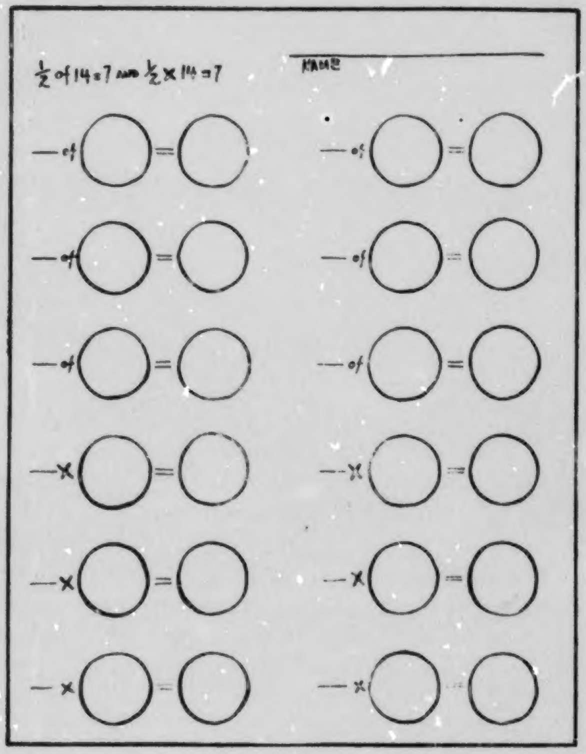
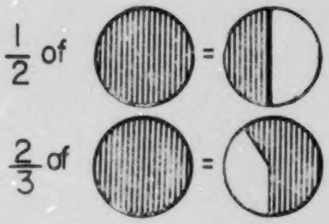

½ of 14 = 7 and ½ × 14 = 7
NAME
— of
— x
=

This report form can be used by the learner to make independent investigations or it can be particularized by the teacher.

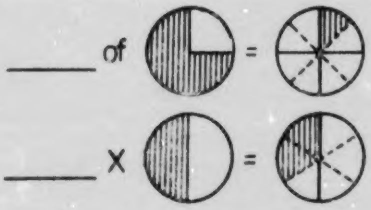
Problems such as the following might be posed on this page.

(1)



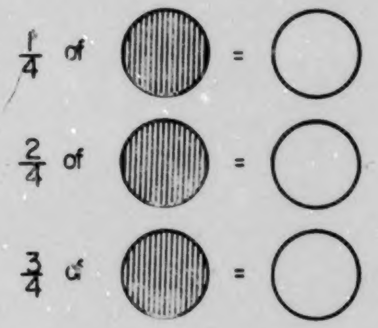
The learner must figure out what part of the first circle must be shaded.

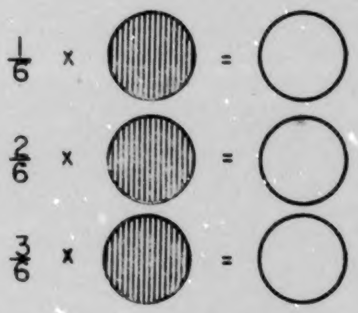
(2)



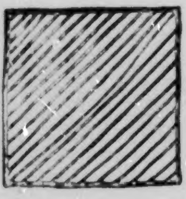
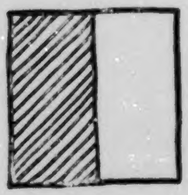
The learner must figure out what fraction can be written on the line to make the statement true.

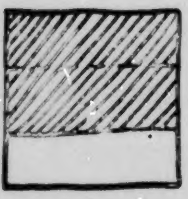
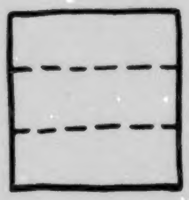
Sequences of 3 examples might be set up to begin building referents for addition of fractions. For example:

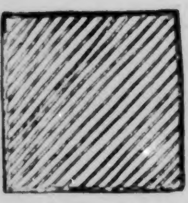
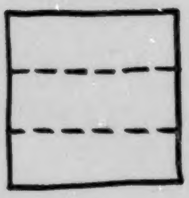

1/4 of
=
2/4 of
=
3/4 of
=

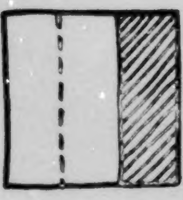
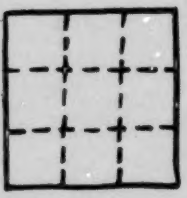


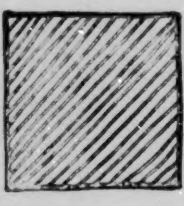
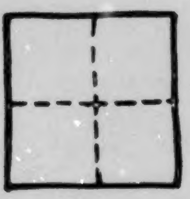
$\frac{1}{2}$ of 10=5 $\frac{1}{2}$ × 10=5

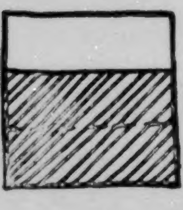
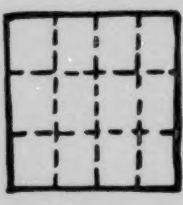
$\frac{1}{2}$ of  = 

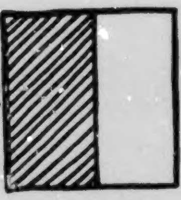
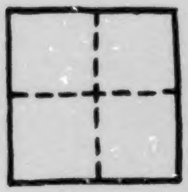
$\frac{1}{2}$ de  = 

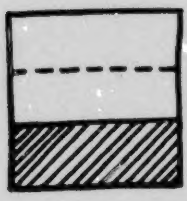
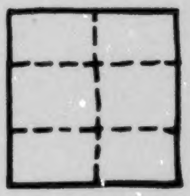
$\frac{2}{3}$ of  = 

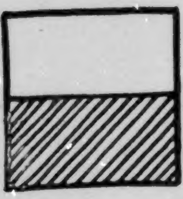
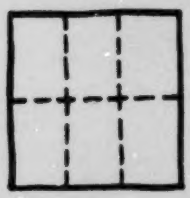
$\frac{2}{3}$ X  = 

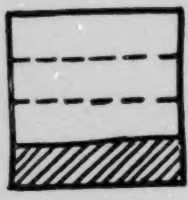
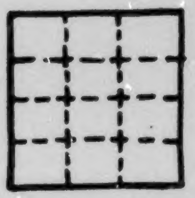
$\frac{3}{4}$ X  = 

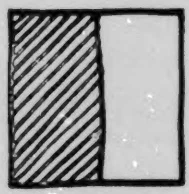
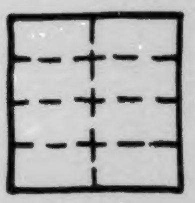
$\frac{3}{4}$ X  = 

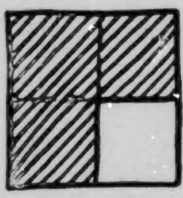
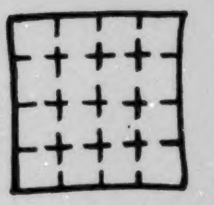
$\frac{1}{2}$ de  = 

$\frac{1}{2}$ of  = 

$\frac{2}{3}$ de  = 

$\frac{2}{3}$ X  = 

$\frac{3}{4}$ X  = 

$\frac{3}{4}$ X  = 

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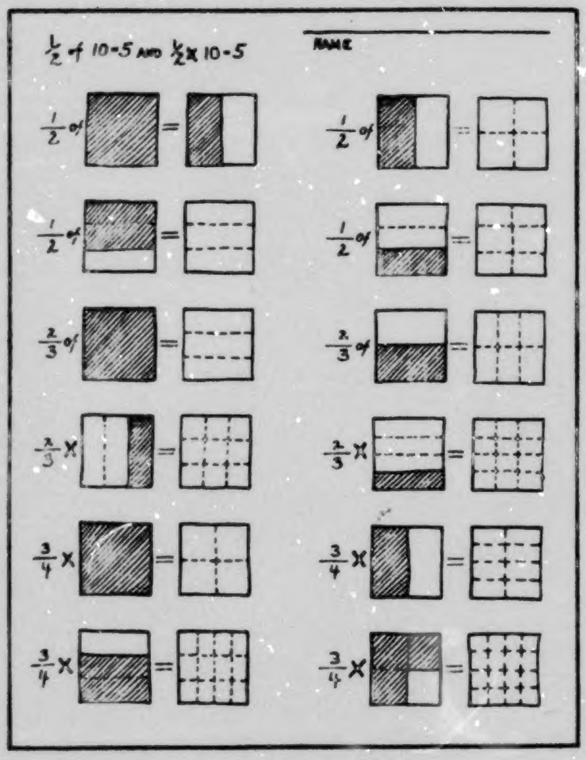
1/2 of 10-5 AND 1/2 x 10-5
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This activity is a continuation of previous activities, with the change that a square is now used instead of a circle. Once again, children should be encouraged to avoid going through two separate steps if they are certain of the outcome.

Lesson XII

I. Measurement (Scaling).

- A.) If the scaling task assigned in the previous lesson was not completed, use this time to do so.
- B.) Discuss other possible scaling projects in two and three dimensions.
- C.) Discuss the relationship of this activity and the idea of a function. Also, develop ideas of ratio and proportion.

II. Logic.

- A.) Play logic games, dealing with numbers, sorting, etc.

III. Numbers and Operations.

- A.) Using Cuisenaire Rods, Wirtz book, review ideas of equivalent fractions.
- B.) Present operations with fractions, using some physical model, such as Cuisenaire Rods.

BRAIN TEASERS

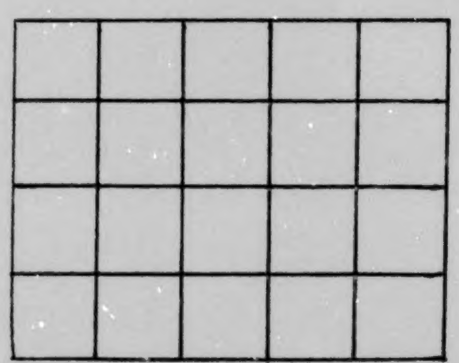
Man, the thinker, is always trying something new and different. Here are some different number problems. Each problem uses the digits 1 to 9. Replace each question mark with a digit. You may not use a digit more than once in a problem. To help you along we have supplied one digit in each problem.

1. ? ÷ ? + ? + ? ÷ ? ÷ ? + ? = ?6
2. ??? + ??? = ??7
3. 9?? - ??? + ???
4. 1?? + ??? = ???
5. ???? x 4 = ????

Numbers can do strange things. They can at times do the impossible, or so it seems. How many of the following strange things can you do with numbers? You may write the numbers in any manner you wish. You may use any symbols with the numbers. However, you may use only the numbers given.

1. Can you make four 2's equal 23?
2. Can you make four 2's equal one?
3. Will three 3's equal 30?
4. Can you make three 2's equal 24?
5. Will three 2's equal 11?
6. Can three 5's equal 30?
7. Will four 6's equal 48?

For this puzzle all you need use are the numbers 1, 2, 3, and 4. Place one number in each box so that the box next to it holds a different number. In other words, a 2 can touch only 1, 3, and 4. Fill all boxes using the numbers 1, 2, 3, and 4 over again and again.



BRAIN TEASERS

Here is a little test of one's ability to use addition, subtraction, multiplication, and division with imagination. We will give you three numbers, an equal sign, and a fourth number. Using any process you wish, but keeping the numbers in the order given, try to have them equal the fourth number.

Perhaps an example will clear up any questions. 8, 3, 3, = 15. Obviously, something special will have to be done to make a correct equation. We suggest this as our answer: 8 - 3 x 3 = 15. Good luck on the ten which follow!

1. 6, 8, 4 = 12
2. 5, 5, 1 = 1
3. 8, 3, 5 = 1
4. 7, 7, 6 = 0
5. 9, 8, 8 = 8
6. 6, 3, 6 = 3
7. 9, 3, 6 = 2
8. 5, 5, 5 = 5
9. 9, 4, 6 = 6
10. 8, 4, 4 = 3

Using the digits in the order given, work out a problem so that the first group of three is equal to the second group of three.

Here is a starter - 9, 6, 3 = 8, 5, 3 Answer = 9 - 6 - 3 = 8 - 5 - 3 = 0

1. 5, 4, 8 = 6, 6, 8 =
2. 9, 8, 5 = 2, 6, 2 =
3. 3, 3, 8 = 6, 1, 8 =
4. 1, 2, 3 = 2, 9, 5 =
5. 9, 2, 4 = 6, 4, 9 =
6. 6, 3, 7 = 9, 3, 8 =
7. 5, 7, 9 = 4, 6, 8 =
8. 4, 9, 6 = 6, 7, 7 =
9. 1, 4, 5 = 9, 8, 1 =
10. 2, 4, 8 = 9, 3, 3 =

In the progressions that follow, try to discover the numbers or letters which should appear in the blank spaces. An example might be: 1 3 5 7 --. It is easy to see that the missing numbers should be 9 and 11. This progression was one of consecutive odd numbers, or odd numbers which appear in order. Most of the following progressions are pretty obvious though one or two may make you use your powers of detection.

1. 2 7 12 - -
2. 1 4 9 16 - -
3. 1 3 6 10 - -
4. 1 3 7 15 - -
5. 1 2 4 8 16 - -
6. 2 3 5 9 17 33 - -
7. 10 9 11 8 12 7 - -
8. 1 2 5 6 9 10 - -
9. 1 3 9 27 - -
10. 1 10 2 9 - -
11. 0 6 13 21 - -
12. 1 2 5 14 41 - -

Student Problem #3

In a town there are seven men, some of whom know each other, and some of whom do not. Below are listed pairs of men who are friends:

Fred knows Sam
John knows Bob
John knows Bill
Fred knows Bob
Fred knows John
Sam knows Bob
Tom knows John
Sam knows Tom
Sam knows John
Sam knows Bill
John knows Andy
Tom knows Bob

(Whenever 'Fred knows Sam,' it is assumed that 'Sam knows Fred' too.)

In the same town there are seven women. Below are listed pairs of women who know each other:

Mary knows Joan
Jane knows Sally
Carol knows Sally
Betty knows Jane
Mary knows Betty
Carol knows Mary
Ann knows Mary
Ann knows Sally
Betty knows Ann
Sally knows Mary
Mary knows Jane
Sally knows Betty

The question is: Can you marry off the men to the women in such a way that whenever two men know each other, their wives will too, and vice versa; and so that whenever two men do not know each other, their wives don't either?

Are there any diagrams you can draw that might help you to solve this problem?

Who Owns the Zebra?

The brain teaser - Who Owns the Zebra - has been challenging the logic and luck of many who have run across it for several years now and it is still appearing in various publications. There has been at least one claim that the puzzle was worked in seven minutes, but you win our congratulations if you can find the answer within the hour. Some lively minds have worked on it all day and given up. So give yourself a pat on the back if you solve it, but don't feel you have to kick yourself if you don't. It's all in fun.

There are five houses, each of a different color and inhabited by men of different nationalities, with different pets, drinks and cigarettes.

The Englishman lives in the red house.

The Spaniard owns the dog.

Coffee is drunk in the green house.

The Ukrainian drinks tea.

The green house is immediately to the right (your right) of the ivory house.

The Old Gold smoker owns snails.

Kools are smoked in the yellow house.

Milk is drunk in the middle house.

The Norwegian lives in the first house to the left.

The man who smokes Chesterfields lives in the house next to the man with the fox.

Kools are smoked in the house next to the house where the horse is kept.

The Lucky Strike smoker drinks orange juice.

The Japanese smokes Parliaments.

The Norwegian lives next to the blue house.

Now who drinks water and who owns the zebra?

TAKE A NUMBER GAME

This is a number puzzle. Try it with several numbers and see if you can discover the trick.

1. Take a number, such as 8.
2. Double it.
3. Add any even number, such as 4, to the number you get.
4. Take away half of what you get in step 3.
5. Take away the number you started with.
6. The result is 2 when 4 is added at step 3.

Try adding 2, 6, 8, or any other even number at step 3 and see what the result is. Is the answer half of the number you add at step 3?

DETECTIVE (Place Value)

__Procedure:__ Be a detective and investigate some missing numerals. Listen carefully for their descriptions.

1. Attention all detectives! We have a missing numeral. He is even. He has an older sister who is four. Can you identify him? (2)
2. Attention detectives! Some numerals are missing. They were last seen around the middle of the number line. They have 4 tens, are odd, and are larger than 33. What are they? (41, 43, 45, 47, 49)
3. All detectives be on the lookout for a missing numeral! His hundred's place is an even number between 2 and 6. His ten's place is 5. His one's place is an odd number less than three. Who is he? (451)

Lesson XIII

I. Numbers and Operations.

   A.) Spend this time in a "fractions' Lab." This lab should contain many activities with several different physical materials. Materials suggested are: Geoblocks, Cuisenaire Rods, Unifix Cubes, Fraction Bars, Pattern Blocks, Tangrams, Discovery Blocks.

II. Measurement and Functions.

   A.) Review area functions and formulas learned in Chapter VI.

   B.) Use unit blocks to develop the formula for the volume of a cube. Do this by collecting data of various sized cubes, looking at the pattern, and deriving the function. From this idea, develop volume formulas for rectangular parallelepipeds.

   C.) Use the Sage Kit to investigate the volumes of other solids.

FRACTIONS (color cubes)

1. Build a rectangle with 2 red and 2 blue cubes. Call this rectangle one. What is the value of:

   1 red cube_____ 1 blue cube_____ 2 red cubes_____

2. Build a rectangle with 3 red, 2 blue and 1 yellow cube. Call this rectangle one. What is the value of:

   1 yellow cube_____ 2 blue cubes_____ 3 red cubes_____

   all blue + all red______ all blue + yellow_____

3. Build a rectangle with 8 cubes of your choice. Call this rectangle one. What is the value of:

   1 cube_____
   2 cubes_____ or _____
   3 cubes_____
   4 cubes_____ or_____ or_____
   5 cubes_____
   6 cubes_____ or _____
   7 cubes_____
   8 cubes_____ or _____ or _____ or _____

4. Build a rectangle with 4 red, 3 yellow, 2 black, 1 white and 2 green cubes (12 cubes altogether). Call this rectangle one. What is the value of:

   White _____ all Black _____ or _____

   all Yellow _____ or _____ all Red _____ or _____

   all Green + all Yellow + White _____ or _____ or _____

   Do the following problems. Use the color cubes to help you.

   EXAMPLE: white + red = 5 cubes   red + yellow = 7 cubes

   1/12 + 1/3 = 5/12   _____ + _____ = 7/12

   green + red = 6 cubes   green + _____ = 3 cubes

   _____ + _____ = _____ or _____   _____ + _____ = 3/12 or _____

   Make up your own addition problems. Be sure to use both cubes and fractions. Call a rectangle made of 12 cubes one (1).

FRACTIONS (Color cubes, cont.)

1. Build a rectangle of 9 cubes, using any colors you wish. Call this rectangle one. What is the value of the following:

1 cube _____

2 cubes _____

3 cubes _____ or _____

4 cubes _____

5 cubes _____

6 cubes _____ or _____

7 cubes _____

8 cubes _____

Do the following problems. Use the color cubes to help.
Example:

3 cubes - 2 cubes = 1 cube

1/3 - 2/9 = 1/9

7 cubes - 5 cubes = 2 cubes

_____ - _____ = _____

6 cubes - 3 cubes = 3 cubes

_____ - _____ = _____ or _____

9 cubes - 6 cubes = 3 cubes

9 cubes - 3 cubes = 6 cubes

1 - 1/3 = _____ or _____

8 cubes - _____ cubes = 3 cubes

_____ - _____ = _____ or _____

Write four problems of your own. (9 cubes in a rectangle = one.)

---

2. Take 6 red, 2 white and 4 black cubes. Build a rectangle.

Red = _____

White = _____

Black = _____

FIND ALL POSSIBILITIES

Find the following: _____ of _____ = _____. What color would the cubes be?

What color is 2/3 of 1/2?

What color is 1/3 of 1/2?

What color is 1 = 1/2?

What color is 1 = 2/3?

FRACTIONS (Color cubes, cont.)

Using 6 red, 2 white and 4 black cubes:

1 ÷ 1/2 - How many 1/2's in one? - How many 1/2's of one (1) in 1/2?

1/2 ÷ 1/3 - How many 1/3 of 1 will you find in 1/2?

1/3 ÷ 1/3; 1/3 ÷ 1/2 What is the question here?

Look at your answers. Can you find a pattern? Can you solve problems without the blocks?

Why is 12 such a nice number with which to do fractions?

What other numbers would be good to use?

1/3 8/9 2/3 1/4	2/11 4/6 5/7 2/9	3/8 15/21 1/6 1/2	4/9 2/12 1/2 3/5
2/8 1/6 4/5 5/6	4/18 8/10 7/9 24/27	2/4 14/18 4/12 1/7	6/10 1/3 2/3 3/8
10/12 9/11 1/5 5/9	8/9 2/10 1/8 12/27	2/14 2/16 5/8 3/4	6/16 10/16 5/16 2/6
15/27 1/5 3/7 3/11	4/9 6/14 4/7 1/4	6/8 8/14 12/24 7/8	1/3 1/2 5/11 7/11

FRACTION PUZZLE. CAN BE RUN OFF ON CONSTRUCTION PAPER -- CUT APART AND HAVE CHILDREN PUT TOGETHER HAVING ALL SIDES MATCHING EQUIVALENT FRACTIONS.

Center for the Improvement of Mathematics Education

In-service Training Workshop

PATTERN BLOCKS

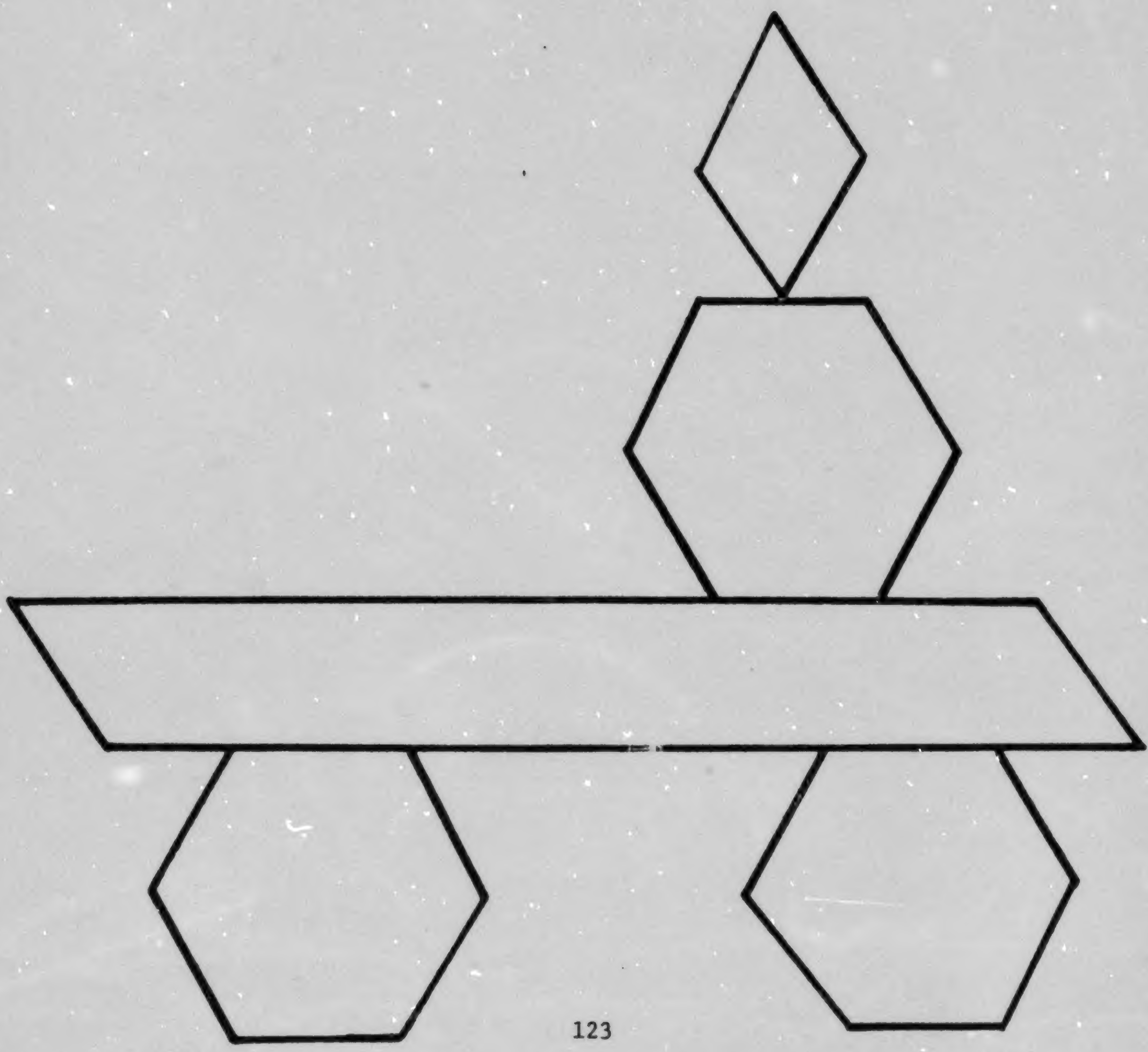
1. Choose a flat space. Make a "floor" of Pattern Blocks over your flat space.
2. Make a design in your "floor."
3. Name the shapes in your design.
4. Choose one block. Make a "floor" around it.
5. Choose another block. Make another "floor" around it.
6. Make a "floor" around each different kind of block.
7. Make a "floor" of only triangles and blue diamonds.
8. Choose two other shapes and make a "floor."
9. Make as many different kinds of "floors" as you can.

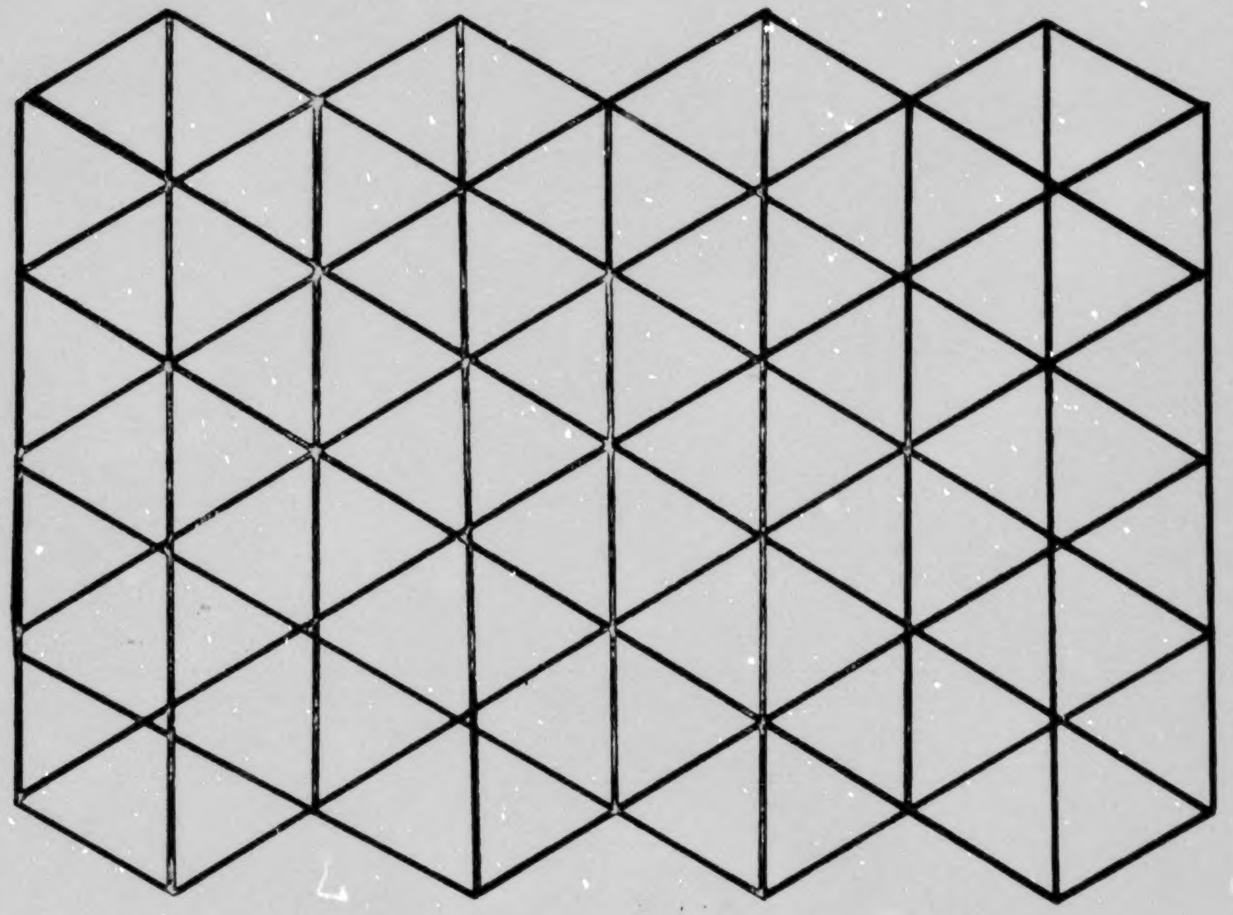
- -

1. Do this with a friend. Each of you make a design with your blocks. Here are some suggestions: people, a castle, animals, a car.
2. Copy each other's designs with blocks.
3. Can you make them exactly alike?
4. Trace around your designs on heavy paper.
5. Give it to a friend to see if he can fill in your outline.
6. Take your friend's outline and see if you can fill it in.
7. Make other outlines. Keep them in a file.

Page 2 - Pattern Blocks

1. Make this train
2. Can you make it using just △'s?
3. Can you make it using just ◇'s?





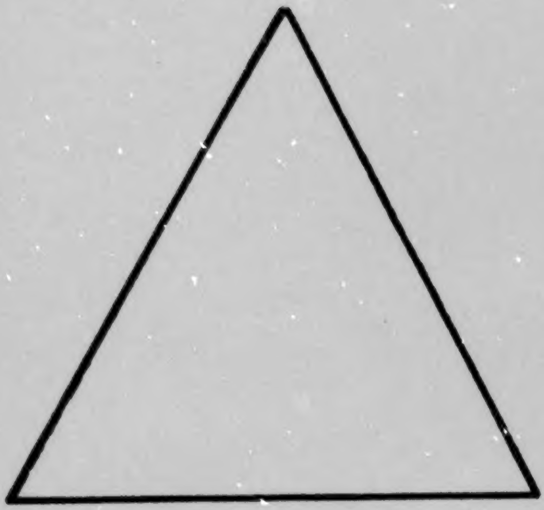
This design was made on isometric paper using triangles (G), blue diamonds (B), trapezoids (R), and hexagons (Y) only. How much is this pattern worth if:

a.	G is 1	e.	G is worth 20¢
b.	B is 1	f.	B is worth 10¢
c.	R is 1	g.	R is worth 45¢
d.	Y is 1	h.	Y is worth 60¢

Make up some of your own.

Build more designs using isometric paper. Color the designs. You might like to bind a collection of designs into a book.

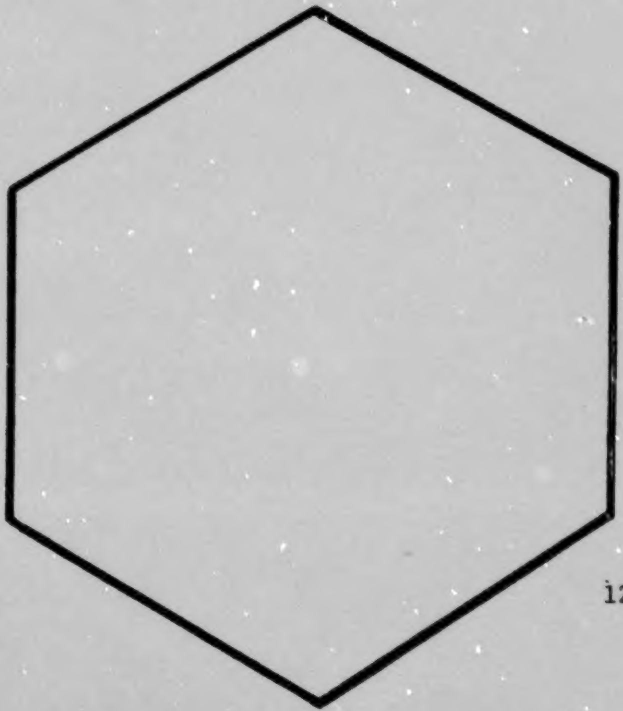
Pattern Blocks



How many different ways can
you fill this shape:

a. with blocks of one color?
b. with blocks of two colors?
c. with blocks of three colors?

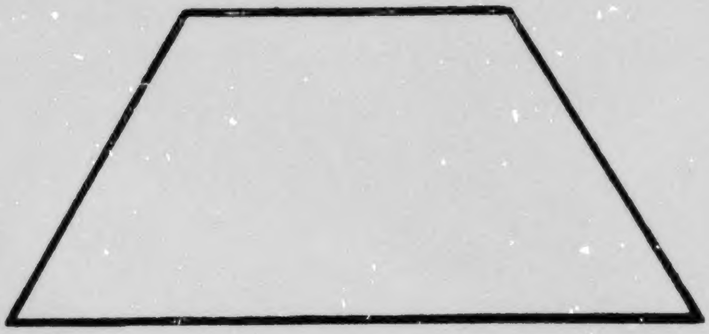
Keep a record.



The least number of blocks it takes to fill this shape is 6 and the greatest number is 24. What other number of blocks can you use?

Keep a record.

(Hint: All numbers are possible. Try to find them.)



Fill this trapezoid.

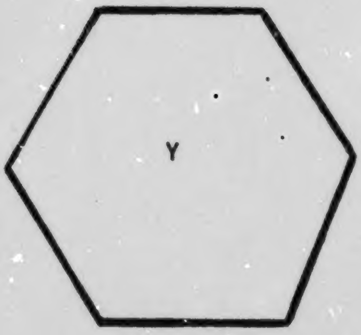
The record below shows how many different shapes were used and the number of each shape needed. Find more ways and record them.

number of different shapes	hexagons	trapezoids	blue diamonds	triangles

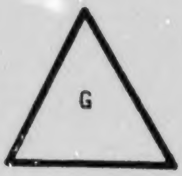
---

1. Trace a shape on isometric paper.
2. Make a record like the one above.
3. See how many different results you can record.
4. Ask someone else to do it and check your results.

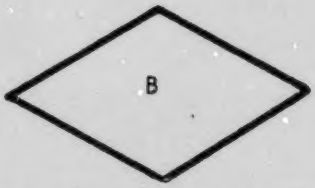
Pattern Blocks

If 

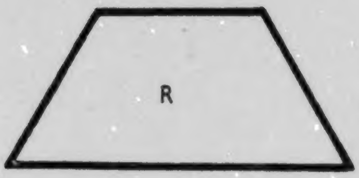
is called 1, then

a. 

is called _________.

b. 

is called ________.

c. 

is called_________.

Let the blue diamond equal 1. What are the others called?

Let the green triangle equal 4. What are the others called?

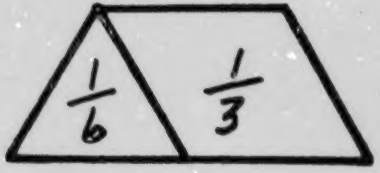
Set the value for any block. Find the value of the others.

Fill in the table

△	◇	⏢	⬡
2	4		
			2
		2	
	5		

Addition

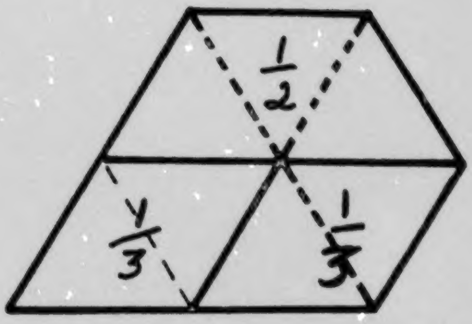
1. $\frac{1}{6} + \frac{1}{3}$ Let Y = 1



G + B = R

$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

2. $\frac{2}{3} + \frac{1}{2}$ Let Y = 1



2 B + R = 7 G or Y + G

$\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ or $1\frac{1}{6}$

3. Try these. Use the blocks to help you.

   a. $\frac{2}{3} + \frac{1}{6}$

   b. $\frac{3}{6} + \frac{1}{3} + \frac{1}{2}$

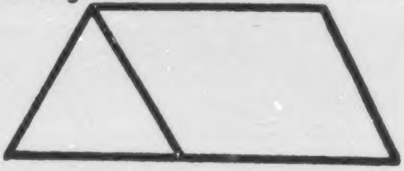
   c. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

   d. $\frac{2}{3} + \frac{5}{6}$

4. Make up some of your own.

Subtraction

1. $\frac{1}{2} - \frac{1}{6}$ Let $Y = 1$. Place the triangle $\left(\frac{1}{6}\right)$ on top of the trapezoid $\left(\frac{1}{2}\right)$. It takes a blue diamond $\left(\frac{1}{3}\right)$ to fill in the remaining space.



R - G = B

$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

2. Try these. Use the blocks to help you.

   a. $\frac{5}{6} - \frac{1}{3}$

   b. $\frac{2}{3} - \frac{1}{6}$

   c. $\frac{2}{3} - \frac{1}{2}$

   d. $1\frac{1}{6} - \frac{1}{2}$

3. Use $>$, =, or $<$

   a. $\frac{1}{2}$ ○ $\frac{1}{3}$

   b. $\frac{1}{6} + \frac{1}{3}$ ○ $\frac{2}{3} - \frac{1}{6}$

   c. $\frac{1}{3} + \frac{1}{2}$ ○ $\frac{4}{6} + \frac{1}{3}$

   d. $1\frac{2}{3} - \frac{5}{6}$ ○ $\frac{1}{3} - \frac{1}{6}$

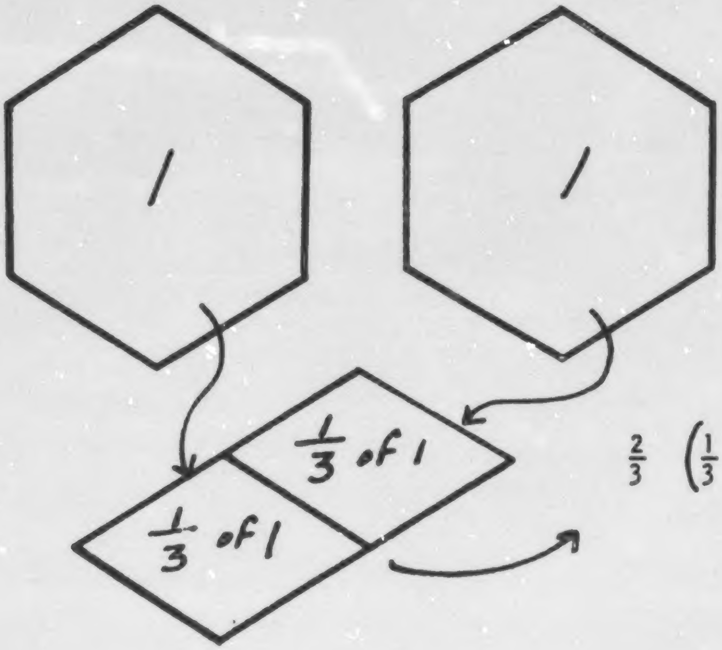
4. Make up some of your own.

Multiplication

1. $\frac{1}{3} \times 2$

$\frac{1}{3}$ of 2

Let Y = 1

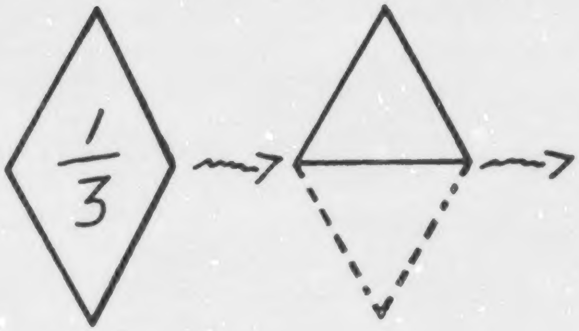


$\frac{2}{3}$ $\left(\frac{1}{3} \text{ of } 2\right)$

2. $\frac{1}{2} \times \frac{1}{3}$

$\frac{1}{2}$ of $\frac{1}{3}$

Let Y = 1



$\frac{1}{6}$ $\left(\frac{1}{2} \text{ of } \frac{1}{3}\right)$

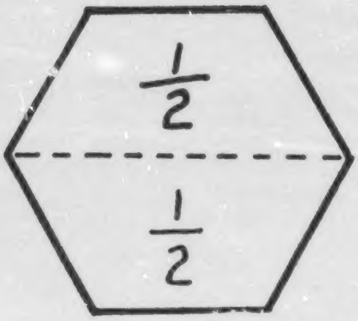
3. Try these. Use the blocks to help you.
   - a. $\frac{1}{2} \times 5$
   - b. $\frac{2}{3} \times 4$
   - c. $\frac{1}{4} \times \frac{2}{3}$
   - d. $\frac{2}{3} \times \frac{1}{2}$

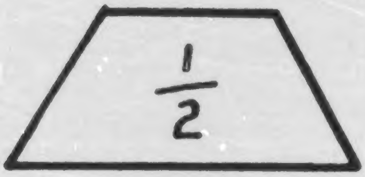
4. Make up some more of your own.

Pattern Blocks

Division

1. $1\frac{1}{2} \div \frac{1}{2}$ Let Y = 1 How many $\frac{1}{2}$'s in $1\frac{1}{2}$?





There are three $\frac{1}{2}$'s in $1\frac{1}{2}$, so

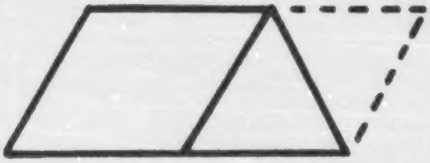
$1\frac{1}{2} \div \frac{1}{2} = 3$

2. $\frac{1}{2} + \frac{1}{3}$ Let Y = 1 How many $\frac{1}{3}$'s in $\frac{1}{2}$?

so $R = \frac{1}{2}$

and

$B = \frac{1}{3}$



There is one $\frac{1}{3}$ and $\frac{1}{2}$ of another $\frac{1}{3}$, so $\frac{1}{2} + \frac{1}{3} = 1\frac{1}{2}$

3. Try these. Use the blocks to help you.

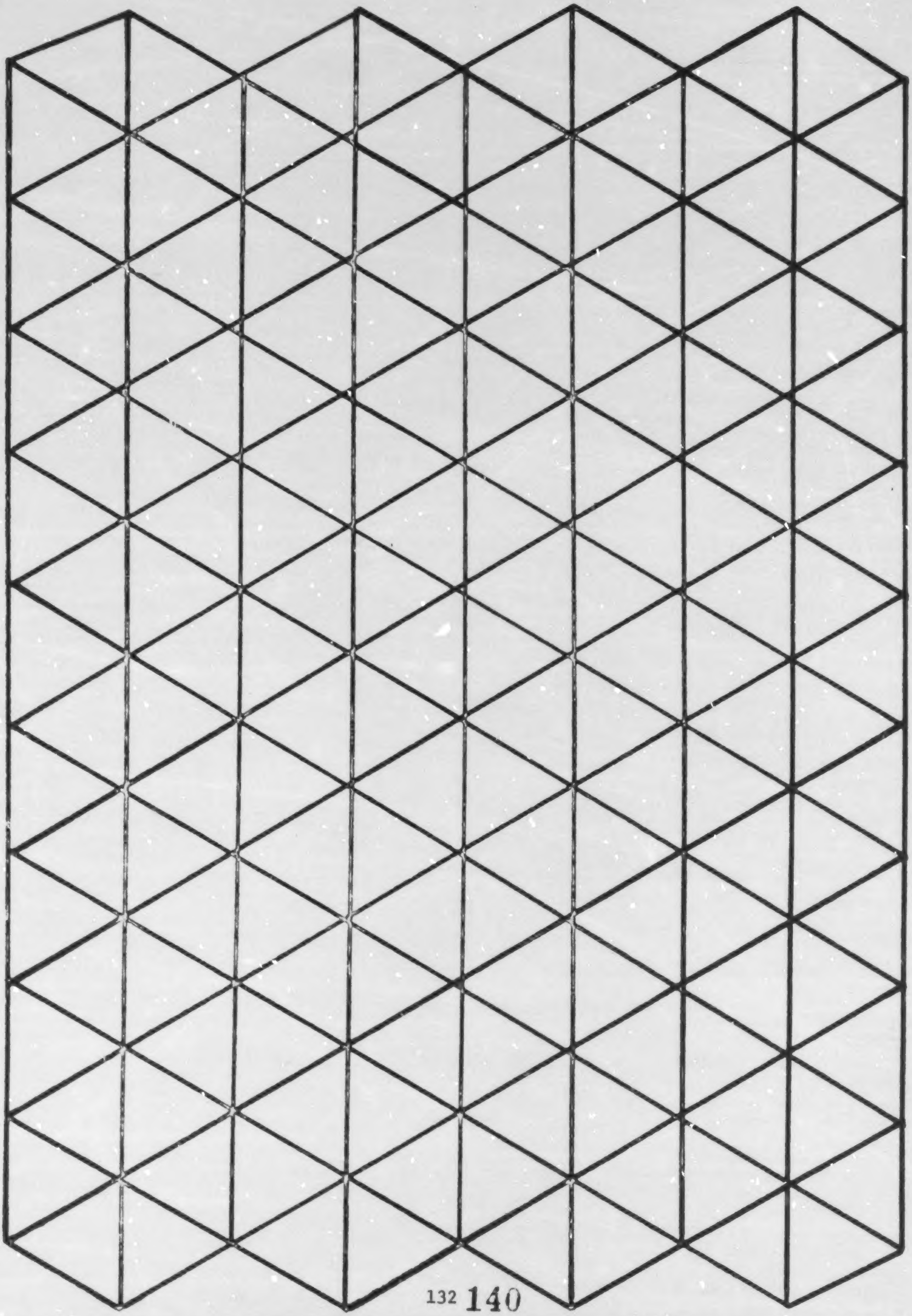
a. $2 + \frac{1}{3}$

b. $\frac{2}{3} + \frac{1}{6}$

c. $3 + \frac{1}{6}$

d. $\frac{5}{6} + \frac{1}{3}$

4. Make up some more of your own.



FRACTIONS (color cubes)

1. Build a rectangle with 2 red and 2 blue cubes. Call this rectangle one. What is the value of:

1 red cube__________ 1 blue cube_________ 2 red cubes____________

2. Build a rectangle with 3 red, 2 blue and 1 yellow cubes. Call this rectangle one. What is the value of:

1 yellow cube__________ 2 blue cubes____________ 3 red cubes___________

all blue + all red______________ all blue + yellow________________

3. Build a rectangle with 8 cubes of your choice. Call this rectangle one. What is the value of:

1 cube_________
2 cubes_________ or _________
3 cubes________
4 cubes________ or ________ or _________
5 cubes________
6 cubes________ or ________
7 cubes________
8 cubes________or________or_______or_______

4. Build a rectangle with 4 red, 3 yellow, 2 black, 1 white and 2 green cubes (12 cubes altogether.) Call this rectangle one. What is the value of:

White________ all Black_________ or _________

all Yellow_________or___________ all Red________or _________

all Green + all Yellow + White_________or _________or ___________

Do the following problems. Use the color cubes to help you.

EXAMPLE: white + red = 5 cubes

$\frac{1}{12} + \frac{1}{3} = \frac{5}{12}$

red + yellow = 7 cubes

___ + ______ = $\frac{7}{12}$

green + red = 6 cubes

_____ + ___ = ______or______

green + ____ = 3 cubes

_____ + ___ = $\frac{3}{12}$ or___________

Make up your own addition problems. Be sure to use both cubes and fractions. Call a rectangle made of 12 cubes one (1).

FRACTIONS (color cubes, continued)

1. Build a rectangle of 9 cubes, using any colors you wish. Call this rectangle one. What is the value of the following:

1 cube______

2 cubes______

3 cubes______or________

4 cubes______

5 cubes________

6 cubes________or________

7 cubes________

8 cubes________

Do the following problems. Use the color cubes to help.
EXAMPLE:

3 cubes - 2 cubes = 1 cube

$\frac{1}{3} - \frac{2}{9} = \frac{1}{9}$

7 cubes - 5 cubes = 2 cubes

______ - ______ = ______

6 cubes - 3 cubes = 3 cubes

______ - ______ = ______or _____

9 cubes - 6 cubes = 3 cubes
9 cubes - 3 cubes = 6 cubes

$1 - \frac{1}{3} =$ ______or______

8 cubes - ______ = 3 cubes

______ - ______ = ______or______

Write four problems of your own. (9 cubes in a rectangle = one.)

---

2. Take 6 red, 2 white and 4 black cubes. Build a rectangle.

Red = __________

White = _________

Black = _________

FIND ALL POSSIBILITIES

Find the following:______of______=______. What color would the cubes be?

What color is $\frac{2}{3}$ of $\frac{1}{2}$?

What color is $\frac{1}{3}$ of $\frac{1}{2}$?

What color is $1 - \frac{1}{2}$?

What color is $1 - \frac{2}{3}$?

Using 6 red, 2 white and 4 black cubes:

$1 \div \frac{1}{2}$ = How many $\frac{1}{2}$'s in one? = How many $\frac{1}{2}$'s of one (1) in $\frac{1}{2}$?

$\frac{1}{2} \div \frac{1}{3}$ = How many $\frac{1}{3}$ of 1 will you find in $\frac{1}{2}$?

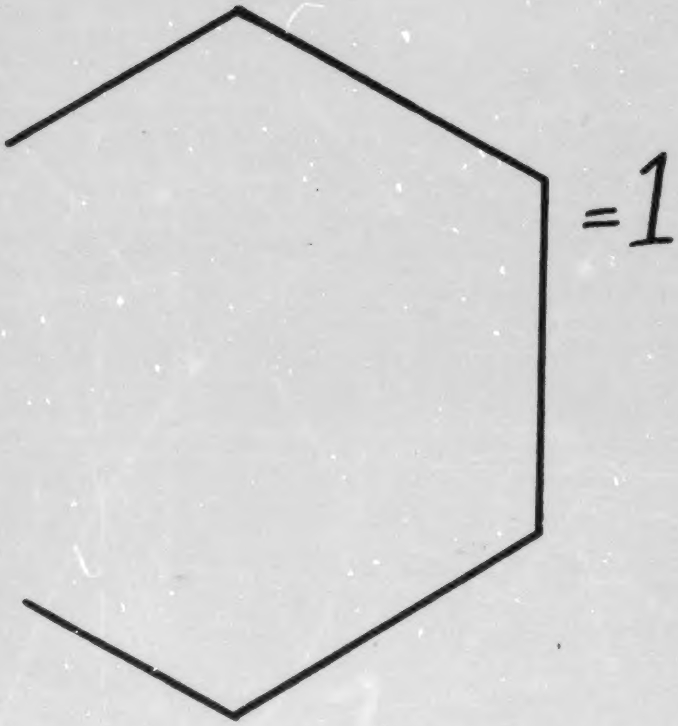
$\frac{1}{3} \div \frac{1}{3}$; $\frac{1}{3} \div \frac{1}{2}$ What is the question here?

Look at your answers. Can you find a pattern? Can you solve problems without the blocks?

Why is 12 such a nice number with which to do fractions?

What other numbers would be good to use?

PATTERN BLOCKS



How many ways can you construct this figure with Pattern Blocks?

Using the blocks, find the fractional part of the following pieces:

Yellow = ________ Red = ________

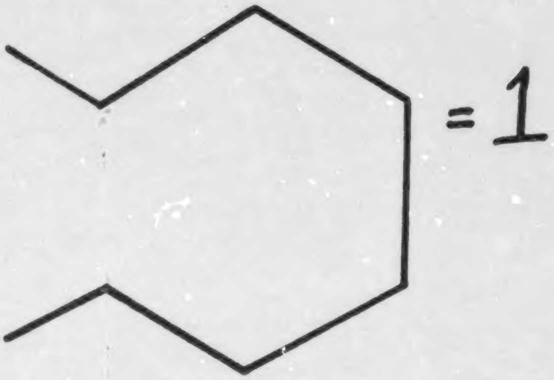
Blue = ________ Green = ________

2 Yellows = ______ 3 Blues = ______

3 Yellows = ______ 5 Greens = ______

2 Blues = ________

Make up some equations of your own like these.



Green = ________

Blue = ________

Red = ________

Yellow = ________

Fish - Red = Yellow:

Yellow - Blue = ____________:

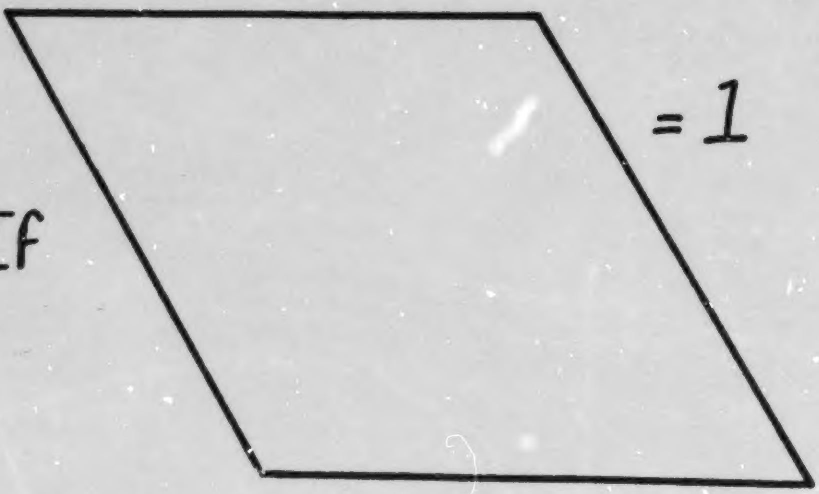
Fish - Blue = ____________:

Red - Blue = ____________:

Make up some subtraction problems of your own. Be sure to use the Pattern Blocks and fractions.

PATTERN BLOCKS

If



= 1

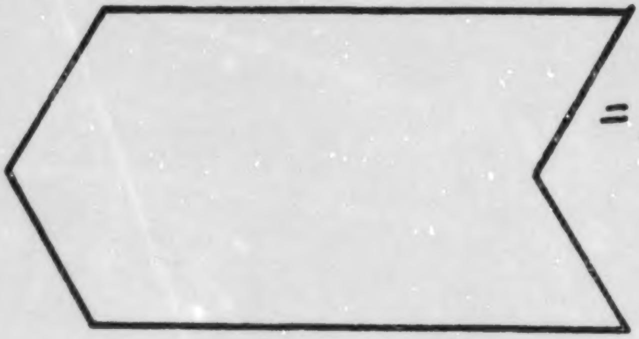
Blue = ________ Red = ________ Green = ________ Yellow = ________

Why does 3 Blue = $\frac{3}{9} = \frac{1}{3}$? Use the pattern blocks to prove your answer.

$\frac{1}{3} + \frac{2}{9} =$ ________ $\frac{2}{3} + \frac{4}{9} =$ ________ $\frac{1}{2} + \frac{1}{3} =$ ________

Prove each of the above by drawing your patterns on paper.

If



= 1

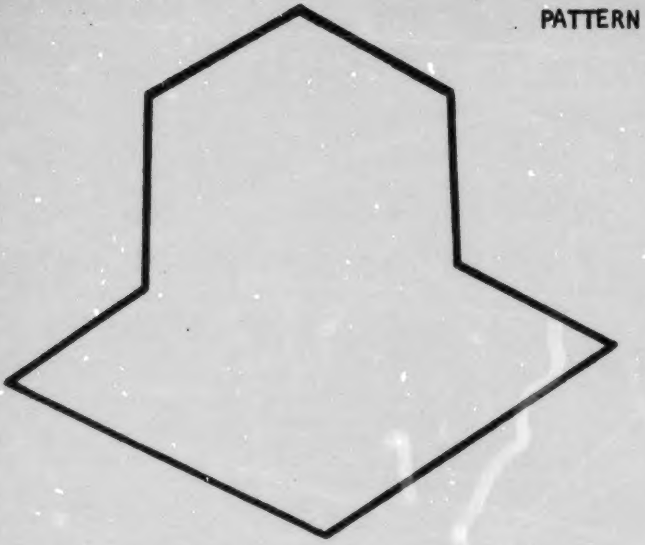
Solve each of the following problems. Prove by drawing your patterns on paper.

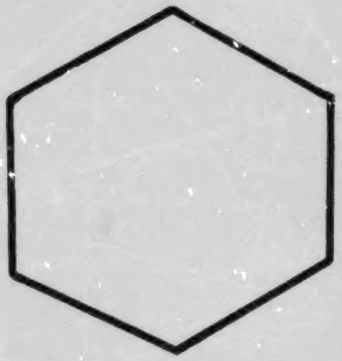
1. Yellow + Red = 1/2 + 1/4 = ________
2. Blue + Red = ________ + ________ = ________
3. Green + Blue = ________ + ________ = ________
4. Yellow + 4 Green = ________ ÷ ________ = ________
5. 2 Blue + 3 Red = ________ + ________ = ________

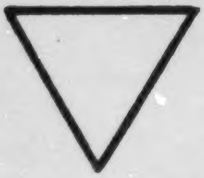
When you add "r" and "r" you get 2r.

What happens when you add "b" and "r"? Why?

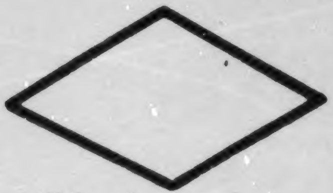
PATTERN BLOCKS


This shape equals 1


This shape equals 1/2


This shape = 1

This shape = 1


This shape = 1

---

2 hexagons = 1

1 hexagon = 1/2

1 trapezoid = ____________

1 blue diamond = ____________

1 triangle = ____________

2 triangles = ___________ or ___________

6 triangles = hexagon; 6/12 = 1/2

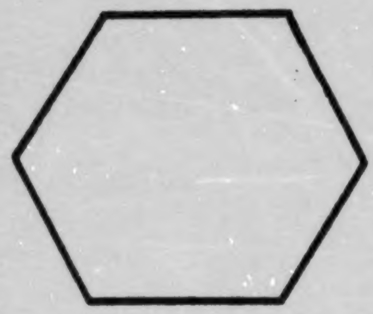
2 trapezoids = hexagon; 2/4 = 1/2

3 blue diamonds + 1 triangle + 1 trapezoid = 10 triangles
3/6 + 1/12 + 1/4 = ____/____

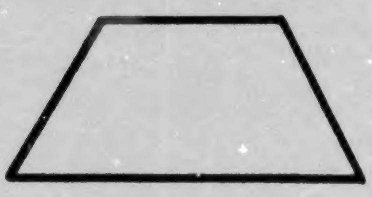
2 triangles + 2 diamonds + _________ = 2 hexagons
_________ + _________ + _________ = _________

1 hexagon + ______ triangles = 3 trapezoids; 1/2 + ____/____ = 3/4
_________ + ______ = ___________

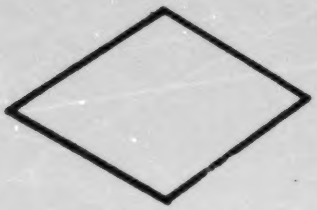
Make up some more true sentences of your own. Remember to use both pattern blocks and fractions.



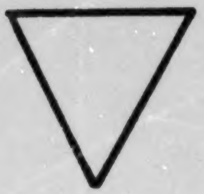
IF HEXAGON = 1,



then TRAPEZOID = ______,



BLUE RHOMBUS = ______,



and TRIANGLE = ______.

---

1. Complete each of the following equations.

__2__ trapezoids = __1__ hexagon: __2/2__ = __1__

______ diamonds = __1__ hexagon: ______ = ______

______ triangles = __1__ hexagon: ______ = ______

______ triangles = __1__ trapezoid: ______ = ______

______ triangles = __1__ blue rhombus: ______ = ______

2. Use both blocks and fractions to solve these problems. Draw your pattern to prove your answer.

2/3 + 1/2 = ______

5/6 + 2/3 = ______

1/2 + 1/3 = ______

5/6 - 1/2 = ______

1 - 1/3 = ______

1 1/2 - 1/6 = ______

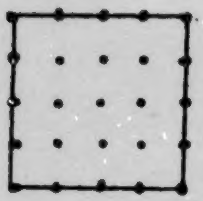
1/2 of 1/2 = ______

1/3 of 1/2 = ______

1/2 ÷ 1/3 = ______ (This is read: How many 1/3's are contained in 1/2? Use blocks to prove.)

5/6 ÷ 1/3 = ______

5/6 ÷ 1/2 = ______



=1

FRACTIONS ON THE GEOBOARD

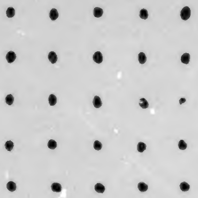
The area of the entire geoboard is called one (1).

What is the value of each small square? (□) ________

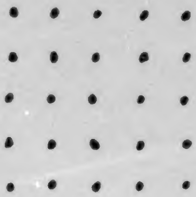
What is the value of 4 small squares? ______________

I. EQUIVALENT FRACTIONS:

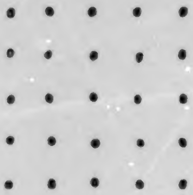
Using your geoboard show an example of each of the values below. Record your results.



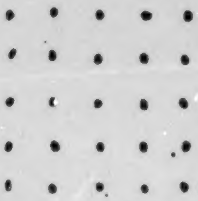
1/2



3/4



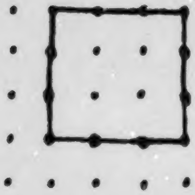
4/8

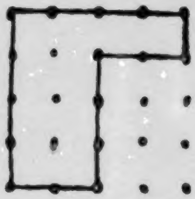


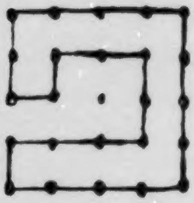
8/16

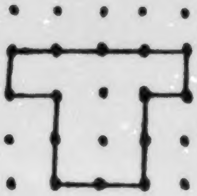
Using your geoboard construct each of the shapes shown below.

Find the value of each shape and record your results.

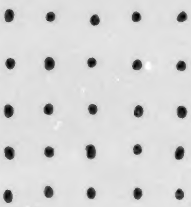


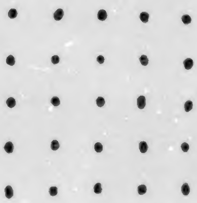


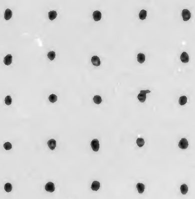


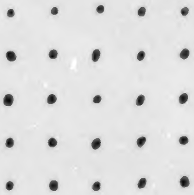


Make up some shapes of your own. Record them on the dot paper and give their value.









Lesson XIV

I. Measurement and Geometry.

A.) Review volume formulas as in Lesson X.

B.) Develop formula for circumference of a circle. This can be done by making a graph showing the circumferences of several circles in relation to their diameters.

II. Numbers and Operations.

A.) Present the problems dealing with long division.

B.) Work on pages 210 through 215 taken from Drill and Practice, Wirtz.

A	x1	x2	x3	x4	x5	x6	x7	x8	x9
B	x10	x20	x30	x40	x50	x60	x70	x80	x90

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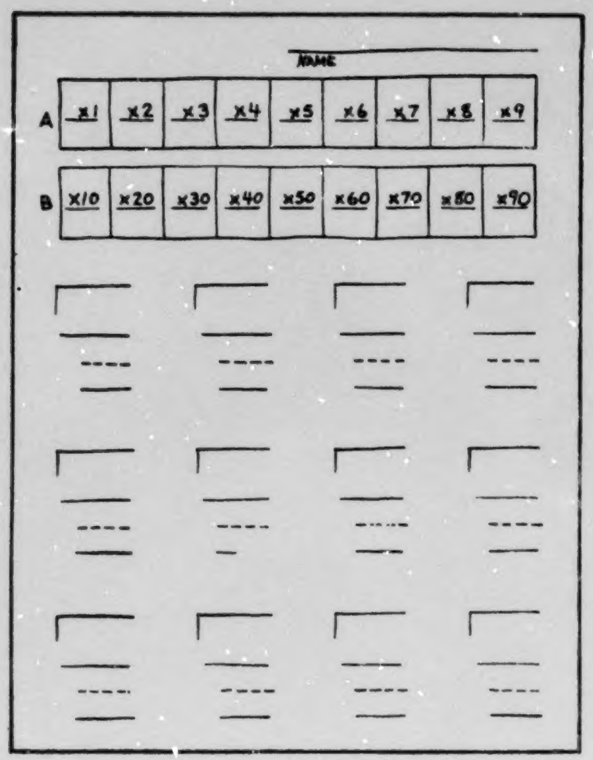

Select any 2 or 3-digit number for the missing factor in the tables and complete those tables—as on the previous pages.

Suppose we select 38 as the missing factor in the tables. If the tables are to be useful, all divisions must be 38. So, the division in each example can be filled in.

$$38\overline{)}$$

The page can be particularized as the previous page by specifying the "final product" or "dividend."

Consider another tactic: "Suppose we want to find numbers we can use so there are no remainders. How should we go about it?"

(Or, perhaps, use the following report form to raise the same question.)

Someone is sure to suggest "multiplying first": __ x 38 = __. The product must be divisible by 38 without any remainder.

This could, of course, be done on scratch paper with the help of the tables:

$$\begin{array}{rl} 38 & \\ \times 47 & \\ \hline 266 & \leftarrow \text{list A}\\ 1520 & \leftarrow \text{list B}\\ \hline 1786 & \end{array}$$

Could we use the form on this page to do the same thing in a different form? One straightforward way is this:

$$\begin{array}{rrl} & 47 & \\ 38 & \overline{)1786} & \\ & 1520 & \leftarrow \text{list B}\\ \cline{2-2} & 266 & \leftarrow \text{list A}\\ & 266 & \end{array}$$

$$\begin{array}{r} 1786\\ \hline +\,1520\\ 266\end{array}$$

Add from bottom to top: 266 + 1520 = 1786, so 1786 must be divided by 38.

Such an activity would give additional insight into the "how" and "why" of the algorism.

If the missing factor selected for the tables is a 3-digit number, nothing new arises—except that the children may have to write smaller numbers in the space provided.

$$\begin{array}{r} 437\\ \times\ 1\\ \hline 437\end{array} \qquad \begin{array}{r} 437\\ \times\ 2\\ \hline 874\end{array} \qquad \begin{array}{r} 437\\ \times\ 3\\ \hline 1311\end{array}$$

and:

$$\begin{array}{rrl} & 32 & \\ 437 & \overline{)13984} & \\ & 13110 & \leftarrow \text{list B}\\ \cline{2-2} & 874 & \\ & 874 & \leftarrow \text{list A}\\ \cline{2-2} \end{array}$$

Of course, if 3-digit answers or "quotients" are worth looking at, then 3 tables may be in order, and more spaces provided in the example forms, but nothing new or interesting will arise. We have the feeling that such straight-forward extrapolations are not very useful in elementary school mathematics.

A	85 ×1	85 ×2	85 ×3	85 ×4	85 ×5	85 ×6	85 ×7	85 ×8	85 ×9
B	85 ×10	85 ×20	85 ×30	85 ×40	85 ×50	85 ×60	85 ×70	85 ×80	85 ×90

$85\overline{)1955}$

B→

A→

$\begin{array}{r} 74 \\ 85\overline{)} \end{array}$

$85\overline{)3060}$

$\begin{array}{r} 85 \\ 85\overline{)} \end{array}$

$85\overline{)4080}$

$\begin{array}{r} 96 \\ 85\overline{)} \end{array}$

$85\overline{)4420}$

$\begin{array}{r} 79 \\ 85\overline{)} \end{array}$

$85\overline{)7310}$

$\begin{array}{r} 78 \\ 85\overline{)} \end{array}$

$85\overline{)8245}$

$\begin{array}{r} 64 \\ 85\overline{)} \end{array}$

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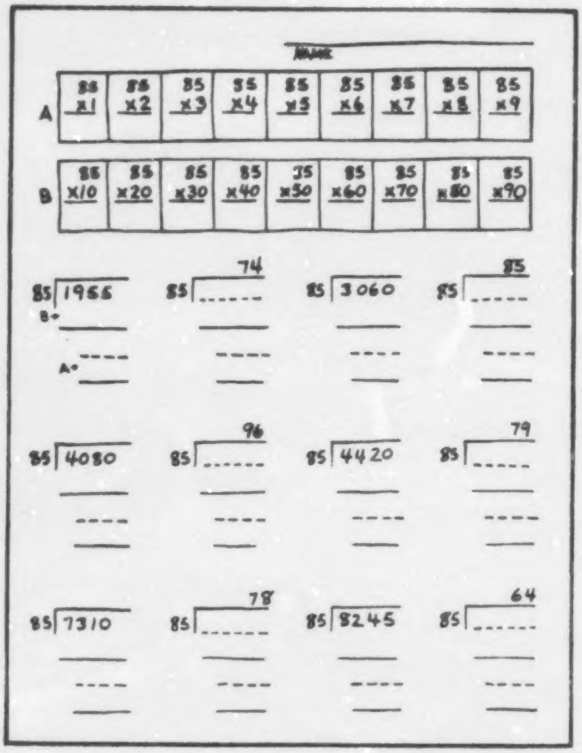


NAME ____________

A	85 x1	85 x2	85 x3	85 x4	85 x5	85 x6	85 x7	85 x8	85 x9
B	85 x10	85 x20	85 x30	85 x40	85 x50	85 x60	85 x70	85 x80	85 x90

$85\overline{)1955}$ B→ A→ $\quad 85\overline{)\;\;\;}$ 74 $\quad 85\overline{)3060} \quad 85\overline{)\;\;\;}$ 85

$85\overline{)4080} \quad 85\overline{)\;\;\;}$ 96 $\quad 85\overline{)4420} \quad 85\overline{)\;\;\;}$ 79

$85\overline{)7310} \quad 85\overline{)\;\;\;}$ 78 $\quad 85\overline{)8245} \quad 85\overline{)\;\;\;}$ 64

Two columns of the examples use the usual long-division format to ask "division" questions. The other columns use the same form but ask "multiplication" questions. Using the same forms and the same tables can you answer both kinds of questions?

$$85\overline{)1955} \text{ and } \overset{74}{85\overline{)\quad\quad}}$$

If you put 1955 books on 85 shelves with the same number of books on each shelf, how many would there be on each shelf?

If you had 85 shelves and 74 books on each shelf, how many books would you have?

How can you use the same tables (multiples of 85) to answer these two different questions?

The flow of these two related algorisms and their close relationships are usually lost in a heavy sea of difficult and incidental computations.

If a child has difficulties with the question of

$$\begin{array}{r}85\\ \times\ 4\\ \hline\end{array} \text{ and } \begin{array}{r}85\\ \times 70\\ \hline\end{array}$$

how could he understand the standard algorism for finding the answer or product:

$$\begin{array}{r}85\\ \times 74\\ \hline\end{array}$$

If a child has difficulty with the questions of

$$\begin{array}{r}85\\ \times\ 8\\ \hline\end{array} \text{ and } \begin{array}{r}85\\ \times 40\\ \hline\end{array}$$

how could he understand the "why" and "how" of the standard algorism for finding the answer or quotient.

$$85\overline{)4080}$$

which when completed in standard form is

$$\begin{array}{r}48\\ 85\overline{)4080}\\ 340\\ \hline 680\\ 680\\ \hline\end{array}$$

If the focus is on the algorism, then remove every distraction and all complexity to let the central problem be the central task.

If our goal is friendliness with numbers, we will remove every possible threat that would distract any child from the particular we are looking at. When he has created necessary multiplication tables in advance, then, and only then for many children, does a division example become a reasonable problem.

Lesson XV

I. Probability.

- A.) Set up a laboratory situation, using task cards involving probability concepts. Discuss results of these investigations. Many task cards can be found dealing with probability in Good Time Mathematics, by Marilyn Burns, published by Addison-Wesley.

II. Geometry and Measurement.

- A.) Use the pattern blocks to develop the concept of angle measurement.

III. Review for Test.

- A.) Positive and Negative Numbers.
- B.) True, False, Open Statements.
- C.) Surface area and volume functions for solids.
- D.) Operations with Fractions.
- E.) Functions.
- F.) Scaling.
- G.) Logic games.
- H.) Multiplication and Division of Whole Numbers.

SIX COIN RIDDLES

It is possible to make each of the amounts below with 6 coins. Figure out how?

GOOD EXERCISE WITH MONEY

AMOUNT	1¢	5¢	10¢	25¢	50¢
$3.00					
$1.30					
$1.25					
$1.51					
$ .60					
$1.56					
$ .42					
$1.10					
$ .30					
$1.15					
$2.60					
$ .06					
$1.40					
$ .67					
$ .95					

Partners____________________

____

1. How big is the package?

Its length is twice its width. length__________
Its height is half its width. width __________
Its length is 25 centimeters. height__________

2. How much does it weigh?

One fourth of the package weighs
200 grams less than one half of
the package. weight__________

3. How much did it cost?

If you buy two of them, you
would get $4.50 change from
a $20 bill. cost____________

4. Who is it for? Who is it from?

Children: Todd, Steve, Mark, Jill

It is not from a boy to the girl. For_____________
It is not from a boy to a boy.
It is not for Todd or Mark. From____________

NAME ____________ ________________________

TASK: You are to use the Classified Section to buy a house, furnish it, a car, and any other recreation vehicles, etc. that you wish. Don't go over the amount given.

Amount you begin with: $100,000

Item and a description of it.	Cost	Page number	Balance

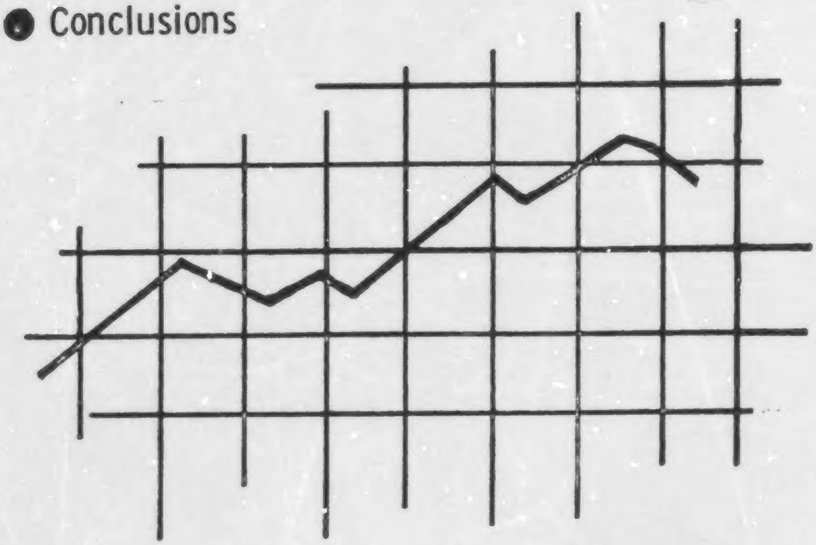
Lesson XVI

I. Examination on materials studied in Lessons IX through XV.

II. Review, discuss any problems the paraprofessional encountered in the examination.

Chapter 6

- Evaluation Design
- Evaluation Instruments
- Analysis and Results
- Conclusions



INTRODUCTORY STATEMENT

The primary purpose of any evaluation design is to provide decision-makers with information which will make their decisions more appropriate. This evaluation report will provide the decision-maker with the information base which was derived from the project entitled Effectiveness of the Paraprofessional on Student Achievement in Mathematics. This information can be used by any decision-maker who is presently using or is planning to use paraprofessionals in the classroom. The decision-maker must be aware of the context from which this information base was derived in order to determine if the information is useful. As is true with all action research projects which are conducted in a social setting, limitations in interpretation and generalization are imposed by the "real world." This report will attempt to provide an objective summary of this particular project and give appropriate cautions to the overinterpretation or misinterpretation of the information presented. Only the decision-maker can determine if this information is appropriate to his situation.

EVALUATION DESIGN

Original Design and Purpose

Illustration 1 provides a visual representation of the evaluation design as it was proposed at the initiation of the project. Each of the column headings will be described and discussed under their respective discussion numbers as indicated on the illustration.

Discussion 4

Discussion 5

Discussion 2

Discussion 3

Discussion 1

GRADE LEVEL	TRAINING LEVEL OF PARA-PROFESSIONAL	P PRO #	MORNING		AFTERNOON	
			SCHOOL 1	SCHOOL 2	SCHOOL 1	SCHOOL 2
4	TRAINED	1	*			*
		2		*	*	
	UNTRAINED	1		*	*	
		2	*			*
	NO					
			*	*	*	*
5	TRAINED	1	*			*
		2		*	*	
	UNTRAINED	1		*	*	
		2	*			*
	NO					
			*	*	*	*
6	TRAINED	1	*			*
		2		*	*	
	UNTRAINED	1		*	*	
		2	*			*
	NO					
			*	*	*	*
7	TRAINED	1	*			*
		2		*	*	
	UNTRAINED	1		*	*	
		2	*			*
	NO					
			*	*	*	*

Illustration 1. Evaluation Design
* indicates assignment to classroom

Discussion 1 - Grade Level

Effectiveness of paraprofessionals was investigated at four different grade levels, grades 4, 5, 6, and 7. Four reasons are given for the choice of these particular grade levels:

1. The investigation should be conducted across multiple grade levels to determine if differential effects exist at different age levels.

2. Elementary and junior high grades should be included to determine if paraprofessionals have a differential affect in the different instructional settings which exist in these two levels within this district.

3. Primary grades were not included because of the choice of the math content emphasis as a curriculum emphasis. It was believed that math was treated as a more formal and unique content emphasis at these grades making it easier to place paraprofessionals into that specific content emphasis.

4. Primary grades were not included because many federal and state programs were being implemented at the primary levels and it was feared that combination of this project with other already existing projects would produce confounding results.

Discussion 2 - Training Level of Paraprofessionals

Three different levels of paraprofessional participation were included within this study.

Level 1 - Paraprofessional in the classroom received training in basic math competencies and instructional techniques for teaching math skills to students in the classrooms.

Level 2 - Paraprofessional in the classroom received no training before being introduced into the classroom.

Level 3 - In these classrooms no paraprofessional assistance was available.

In all classrooms the final decision for how best to utilize the paraprofessional was left to the classroom teacher. The teacher was aware of whether or not the paraprofessional was receiving training.

Discussion 3 - Number of Paraprofessionals

A minimum of two paraprofessionals were included at every grade level and at every level of training. This allowed for more than one school to be used in the experiment at every grade level. This was to assure that if effects were found within any particular grade or level of training, these effects would not be due to one particular paraprofessional or one particular school environment.

Discussion 4 - Number of Schools

A minimum of two schools were used for every combination of training level, grade, level, and time of instruction. Again, this was to eliminate the possibility of a particular school environment being a viable explanation for any effects discovered.

Discussion 5 - Time of Day for Instruction

The original design controlled for time of day for math instruction by having a paraprofessional assigned to morning and afternoon sessions in different schools. Therefore, time of instruction could not be a viable explanation for any effects discovered.

General Comments on Design

As can be seen from Illustration 1 the original intent of the design was to have one classroom with each training level at each grade level within a school. All assignments were made randomly. These assignments were:

1. Paraprofessionals to training levels
2. Paraprofessionals to classrooms
3. Classrooms to time of instruction

The "Real" Design

As is typical in school settings, many variables impose limitations on research designs. Such is the case in this study. Several adjustments were necessary in the three years of project implementation. The major adjustments will be discussed under the same five discussion areas indicated in Illustration 1.

Grade Level

No adjustments were necessary in the grade levels studied in the three years of project implementation.

Training Level of Paraprofessionals

Two major adjustments were made in this area. One adjustment concerned the definition of training. The second adjustment concerned the assignment of paraprofessionals to training levels during the second and third year of the project.

The major training emphasis of the first year of the project for training paraprofessionals was development of math competencies. Some instructional strategies were included in the training. The major emphasis during the second year of the project was development of instructional strategies and materials. Training was minimal during the third year of the project to determine if trained paraprofessionals would retain the competencies developed during the first two years of the project.

Assignment of paraprofessionals to level of training during the first year was random. However, there was some attrition during the second and third year of the project. Because of the training which had been conducted, attrition of trained paraprofessionals created a problem. Two solutions for replacing trained paraprofessionals were considered. One was to have remedial training for replacement trained paraprofessionals. This solution was not possible as the training was already planned for the second year. The second alternative was to select replacement paraprofessionals who had achieved the competencies of the trained paraprofessionals through avenues outside of the project (college training, in-service, etc.). The second alternative was chosen. Replacement paraprofessionals for untrained paraprofessionals were selected in the same way as in the first year. The number of paraprofessionals who left the project each year are indicated in Table 1.

Table 1
Number of Paraprofessionals Leaving Project

Training Level	Original Number of Paraprofessionals	Number of Paraprofessionals Leaving Project	
		End of Project Year 1	End of Project Year 2
Trained	8	5	3
Untrained	8	4	2

Number of Paraprofessionals

No adjustments were made in this area. A minimum of two paraprofessionals were used at each training level in each grade.

Number of Schools

A minimum of two schools were used at each grade level. In fact, during the first year of the project, four schools were used at the elementary level and two schools at the junior high level. Travel time between schools made it prohibitive for paraprofessionals to travel between schools during the second and third year of the project. Paraprofessionals operated in two classrooms in the same school for the second and third year. Because of number of classrooms available, it was not possible to have each training level in at least two schools at each grade level. A fifth school was added at the elementary level to afford more classrooms.

Time of Day for Instruction

No difference was found between morning and afternoon instruction in the first project year. This variable was eliminated as a control variable for the second and third year as most teachers preferred to have math instruction in the morning.

General Comments on Adjustments

All adjustments that were made to the original design compromised the design to some extent. However, all crucial variables were still controlled to a great degree.

Two major points need to be emphasized:

1. Each year cannot be considered as an exact replication of the previous year because of these adjustments. The original intent of the project was maintained.

2. The effects of the paraprofessionals were not additive students across years, i.e., specific students may have a trained paraprofessional one year and no paraprofessional the following year. Therefore, each project year should be considered unique.

EVALUATION INSTRUMENTS

Five instruments were used in the evaluation of this project. These instruments were:

1. California Test of Basic Skills (CTBS) Forms Q and R, 1969 Edition, Mathematics Subtests

2. Semantic Differential Attitude Assessment (SDAA) District developed attitude assessment toward mathematics (See Appendix A and B)

3. Paraprofessional Time Utilization Survey (PTUS) Project developed form for documentation of paraprofessional time utilization (See Appendix C and D)

4. Teacher Survey (TS) Project developed questionnaire for assessing teacher perceptions of paraprofessionals and project (See Appendix E)

5. Paraprofessional Survey (PS) Project developed questionnaire for assessing paraprofessional perceptions of teachers and project (See Appendix F)

Copies of instruments 2 through 5 are included in Appendix of this guidebook.

Each of these instruments will be discussed using four major topic headings.

Rationale for selection of this instrument as an evaluative device in this project

Reliability of the instrument as stated by the publisher or as determined by analysis

Validity of the instrument as stated by the publisher or as determined by analysis

Assessment Timeline indicating when the instrument was used within the project.

California Test of Basic Skills, Mathematics Subtests

Rationale: The CTBS was the state-mandated test of achievement in the State of California at the initiation of this project. Since that time the instrument has been replaced with other assessment strategies in the State. However, the instrument was retained in this project to provide a stable data set across the three years of the project. The mathematics subtests were used as Paraprofessionals were used in conjunction with mathematics instruction in the classroom. Two questions were being investigated using this data set:

1. Will utilization of paraprofessionals in the classroom result in more achievement in the content area by students?
2. Will training of paraprofessionals in math skills result in more achievement in the content area by students?

Reliability: Internal consistency reliability coefficients in excess of .90 are reported in the test manual for each of the subtests in mathematics.

Validity: The State of California had determined that the test was valid for the assessment of mathematics skills by mandate.

Assessment Timeline: Alternative forms of the instrument were used as pre and post tests each project year. Interim data was collected each year; however, this data was based on a limited sample and is not included in this report. The third project year is an exception to this as the interim testing was conducted on a 60% sampling and is used as the post test for the third project year.

Semantic Differential Attitude Assessment

Rationale: The project expectation was that paraprofessionals in the classroom would have an impact on both achievement and attitude of students. This instrument had been developed in an earlier ESEA Title III project within the district. Extensive investigation and revision of the instrument had taken place during that project's implementation. Two questions were being investigated using this data set:

1. Will utilization of paraprofessionals in the classroom result in more improved attitudes toward mathematics by students?

2. Will training of paraprofessionals result in more improved attitudes toward mathematics by students?

Reliability: Internal consistency reliability coefficients ranging from .87 to .97 were obtained in data from this project.

Validity: Validity of the instrument has been supported in four different ways as recommended by Anastasi (1976).

Internal Consistency: As reported above, internal consistency coefficients ranged from .87 to .97.

Sensitivity to Treatment: Attitudes toward mathematics were changed through instructional intervention. Significant differences at the .05 level were found between pre and post tests with instruction intervening.

Discriminant Validity: Teachers selected two groups of students, one group with positive attitudes toward mathematics, the other with negative attitudes toward mathematics. The instrument appropriately discriminated between these two groups. Differences were significant at the .01 level.

Convergent Validity: The instrument should correlate to a greater degree with assessments of similar constructs than with assessments of dissimilar constructs. The instrument was correlated with teachers' ratings of attitudes and with mathematics achievement scores. The corre-

Semantic Differential Attitude Assessment (cont.)

Validity: (cont.)	lations between instrument scores and teachers' ratings were significantly higher than correlations between instrument scores and achievement scores in all instances.
Assessment Timeline:	The instrument was administered as a pre and post test each project year. Interim data was collected each year, however, this data was based on a limited sample and is not included in this report. The third project year is an exception to this as the interim testing was conducted on a 60% sampling and is used as the post test for the third project year.

Paraprofessional Time Utilization Survey

Rationale: The ultimate decision for how the paraprofessional was used in the classroom was determined by the teacher. As this was the case, documentation was needed for how the paraprofessional's time was utilized in the classroom. This information was needed to aid in the interpretation of any effects found in the project. These records were kept by the paraprofessional and consisted of the amount of time spent in specified activities.

Reliability: Intermittent observations were made by the Project Manager in the classrooms to determine the accuracy of the paraprofessional's recording.

Validity: The recording form was based on major categories of activities which could be engaged in by paraprofessionals in the classroom. These categories were based on observations of the classroom and on teacher interviews. Given that these categories were appropriate, observations were made by the Project Manager to determine accuracy of the recording.

Assessment Timeline: This documentation was recorded daily.

Teacher Survey

Rationale:	Information was needed concerning teachers' attitudes and opinions about paraprofessionals in the classroom. This information would aid in the interpretation of any effects found. Also, it could be determined if teachers perceived trained paraprofessionals differently from untrained paraprofessionals.
Reliability:	No one construct was being assessed with this instrument, therefore, internal consistency indices were inappropriate. No other assessments of reliability were conducted.
Validity:	Questions on the survey were based on Project Manager judgments and interviews with teachers and paraprofessionals. Each item assessed its own unique construct.
Assessment Timeline:	The instrument was administered at the conclusion of each project year.

Paraprofessional Survey

Rationale: Information was needed concerning paraprofessional's attitudes and opinions about teachers and the project. This information would aid in the interpretation of any effects found.

Reliability: No one construct was being assessed with this instrument, therefore, internal consistency indices were inappropriate. No other assessments of reliability were conducted.

Validity: Questions on the survey were based on Project Manager judgments and interviews with teachers and paraprofessionals. Each item assessed its own unique construct.

Assessment Timeline: The instrument was administered as a pre and post assessment during the first and second project year.

ANALYSIS AND RESULTS

Data derived from the instruments described in the previous section were analyzed in various ways so that appropriate conclusions could be drawn concerning the results of this project. These analysis procedures and the obtained results will be discussed under the corresponding instrument heading. Discussion will necessarily be brief and limited to major findings using each data set.

California Test of Basic Skills

Analyses: It was necessary to use two analysis procedures because of differences in data types. Two data types were analyzed: raw score gains within project years and grade equivalent gains across project years.

An analysis of variance was conducted at each grade level comparing pre to post test math achievement raw score gains within each project year of students in classrooms with trained paraprofessionals, untrained paraprofessionals, and no paraprofessionals. These same three groups were compared across the three project years using the Kruskal-Wallis H test.

Results: The four analyses of variance, one for each grade level, conducted on achievement gains during each project year were inconsistent in their results. Statistically significant differences were found among the three groups in almost every comparison. However, when post hoc procedures were applied, no interpretable trend could be determined as to whether one group made more achievement growth than the others. It did appear that differences were usually in favor of classrooms with paraprofessionals. In any one project year one could not conclude that utilizing trained paraprofessionals in the classroom consistently resulted in greater math achievement by students in those classrooms.

Decision makers experienced in educational settings could easily understand this finding since they are aware of the numerous variables which can influence the educational setting in any one year. They would expect, however, to see trends across several years if the variable being investigated was effective. For this reason a second analysis was conducted comparing gains made in terms of grade equivalents across all grade levels and across the three years of the project. Because of certain characteristics of this type of data, it was not appropriate to use analysis of variance procedures. The Kruskal-Wallis H test is an appropriate technique to use. In essence this

California Test of Basic Skills (cont.)

Results: (cont.) procedure ranks all gains made by each of the three groups and statistically compares the sum of the ranks for each group. A higher sum of ranks for a particular group would indicate that this group made consistently higher gains than the other groups. Using this analysis procedure, a statistically significant difference at the .05 level was found among the groups. Table 2 reports the grade equivalent gains made by each treatment group for each of the project years within each grade level. The corresponding rank of each gain is also reported. The sum of the ranks for each group is reported in the bottom row of the table.

Table 2

Grade Equivalent Gains and Corresponding Ranks
For Treatment Groups Across Grade Levels and Across Project Years

Grade Level	Project Year	Treatment Groups					
		Trained		Untrained		No Paraprofessional	
		G.E. Gain	Rank	G.E. Gain	Rank	G.E. Gain	Rank
4	1973-1974	1.0	21	1.0	21	.7	9
	1974-1975	2.0	33	.8	13.5	.5	5
	1975-1976	.8(1.3)*	29	.7(1.1)	25.5	.6(1.0)	21
5	1973-1974	.9	16.5	.7	9	.6	6
	1974-1975	.9	16.5	.7	9	1.0	21
	1975-1976	.6(1.0)	21	1.8(2.9)	36	.2(.3)	3.5
6	1973-1974	1.0	21	.7	9	.8	13.5
	1974-1975	1.2	27.5	.7	9	.8	13.5
	1975-1976	.7(1.1)	25.5	.2(.3)	3.5	-.4(-.6)	1
7	1973-1974	2.1	34	1.5	31.5	2.2	35
	1974-1975	1.2	27.5	1.4	30.	1.0	21
	1975-1976	1.5(2.4)	31.5	.1(.2)	2	.5(.8)	13.5
SUM OF RANKS			304.0		199.0		163.0

*Numbers in parentheses are third year gains adjusted because of fewer months between pre and post test.

The sum of ranks for the three groups would indicate that, in the long run, trained paraprofessionals in the classroom resulted in the greatest achievement growth while untrained paraprofessionals in the classroom resulted in greater achievement when compared to classrooms with no paraprofessionals.

Semantic Differential Attitude Assessment

Analyses: Similar to the CTBS analysis, an analysis of variance was conducted at each grade level comparing pre and post test attitudinal improvement across each project year for students in classrooms with trained paraprofessionals, untrained paraprofessionals, and no paraprofessionals. These same three groups were compared across the three project years using the Kruskal-Wallis H test.

Results: The results found were very similar to results found in the previous section, i.e., no consistent trends were found within any particular project year. Results did tend to favor classrooms with trained and/or untrained paraprofessionals. The Kruskal-Wallis H test did indicate a significant difference at the .10 level among groups. Table 3 reports the raw score gains made by each treatment group for each of the project years within each grade level. The corresponding rank of each gain is also reported. The sum of the ranks for each group is reported. The sum of the ranks for each group is reported in the bottom row of the table.

Table 3

Raw Score Gains and Corresponding Ranks for Treatment Groups Across Grade Levels and Across Project Years

Grade Level	Project Year	Treatment Groups					
		Trained		Untrained		No Paraprofessional	
		Gain	Rank	Gain	Rank	Gain	Rank
4	1973-1974	-.5	17	1.1	26.5	-1.6	8
	1974-1975	-.6	14.5	1.3	28	-2.8	4
	1975-1976	-.3	19	3.0	33	3.5	34
5	1973-1974	2.6	31	-1.0	10	-1.7	7
	1974-1975	1.6	29	.6	25	.0	20
	1975-1976	1.9	30	4.6	36	-2.9	3
6	1973-1974	-.5	17	2.9	32	-3.7	1
	1974-1975	.1	21.5	-1.4	9	-2.3	6
	1975-1976	.5	24	3.9	35	-.9	12
7	1973-1974	.1	21.5	-.9	12	-.5	17
	1974-1975	.4	23	1.1	26.5	-.6	14.5
	1975-1976	-3.1	2	-2.6	5	-.9	12
SUM OF RANKS			249.5		278.0		138.5

The sum of the ranks for each group would indicate little difference between positive attitude change in classrooms with trained and/or untrained paraprofessionals. However, classrooms with paraprofessionals appear to have more positive attitude change than classrooms with no paraprofessionals.

Paraprofessional Time Utilization Documentation

Analysis: Percent of time spent in each of the major categories by each of the three groups was calculated monthly for each grade level.

Results: In the elementary grades trained paraprofessionals were used in more instructional roles than untrained paraprofessionals. The untrained paraprofessional was used in more clerical roles than the trained paraprofessional. These differences decreased across the project years. At the junior high level these differences were not as predominant. Both trained and untrained paraprofessionals were utilized predominantly in clerical roles indicating a different environment at this level.

Teacher Survey and Paraprofessional Survey

Analysis: Frequencies or responses were tabulated for each of the items on both surveys. Written comments were summarized and recorded. Many of these comments are included in this guidebook.

Results: Both teachers and paraprofessionals were overwhelmingly in favor of teacher-paraprofessional teams in the classroom. Teachers with paraprofessionals expressed strong objections to losing their paraprofessionals. Many expressed a need for joint training sessions. This was supported by the comments of a few paraprofessionals indicating frustration when they were not allowed to use newly developed competencies in math instruction by their assigned teachers. Untrained paraprofessionals indicated a need for training. Other information from these surveys are presented in other sections of this guidebook.

CONCLUSIONS

1. Within this project, training of paraprofessionals in math skills did result in higher math achievement of students in classrooms into which the paraprofessionals were incorporated.

2. Within this project, students in classrooms which utilized paraprofessionals in math instruction did achieve better in math than students in classrooms without paraprofessionals.

3. Within this project, students in classrooms which utilized paraprofessionals in math instruction did demonstrate more positive change in attitudes toward math than students in classrooms without paraprofessionals.

4. Within this project, trained paraprofessionals were utilized differently from untrained paraprofessionals by teachers at the elementary level.

5. Within this project, teachers were overwhelmingly in favor of paraprofessionals in the classrooms.

APPENDIX A

LA MESA-SPRING VALLEY SCHOOL DISTRICT
INSTRUCTIONAL DIVISION
EDUCATIONAL SERVICES

SEMANTIC DIFFERENTIAL ATTITUDE SURVEY
"Effectiveness of Paraprofessionals"
ESEA TITLE IV, PART C

1975-1976

MATHEMATICS

	1	2	3	4	5	
FUN	😃	🙂	😐	🙁	☹	NOT FUN
HARD	☹	🙁	😐	🙂	😃	EASY
NEATO	😃	🙂	😐	🙁	☹	REALLY DUMB
HAPPY	😃	🙂	😐	🙁	☹	UNHAPPY
NOT CINCHY	☹	🙁	😐	🙂	😃	CINCHY
UNINTERESTING	☹	🙁	😐	🙂	😃	INTERESTING
LOVE	😃	🙂	😐	🙁	☹	HATE
GREAT	😃	🙂	😐	🙁	☹	LOUSY

MATHEMATICS

		1	2	3	4	5	
9.	NO WAY	☹	🙁	😐	🙂	😀	RIGHT ON
0.	UNDERSTAND	😀	🙂	😐	🙁	☹	DO NOT UNDERSTAND
.	UNEXCITING	☹	🙁	😐	🙂	😀	EXCITING
.	BAD	☹	🙁	😐	🙂	😀	GOOD
.	DUMB	☹	🙁	😐	🙂	😀	SUPER
	GET IT	😀	🙂	😐	🙁	☹	DON'T GET IT
.	DISLIKE	☹	🙁	😐	🙂	😀	LIKE
	WOW	😀	🙂	😐	🙁	☹	YUK

APPENDIX B

SEMATIC DIFFERENTIAL ATTITUDE SURVEY

MATHEMATICS

1.	FUN	+2	+1	0	-1	-2	NOT FUN
2.	HARD	-2	-1	0	+1	+2	EASY
3.	NEATO	+2	+1	0	-1	-2	REALLY DUMB
4.	HAPPY	+2	+1	0	-1	-2	UNHAPPY
5.	NOT CINCHY	-2	-1	0	+1	+2	CINCHY
6.	UNINTERESTING	-2	-1	0	+1	+2	INTERESTING
7.	LOVE	+2	+1	0	-1	-2	HATE
8.	GREAT	+2	+1	0	-1	-2	LOUSY
9.	NO WAY	-2	-1	0	+1	+2	RIGHT ON
10.	UNDERSTAND	+2	+1	0	-1	-2	DO NOT UNDERSTAND
11.	UNEXCITING	-2	-1	0	+1	+2	EXCITING
12.	BAD	-2	-1	0	+1	+2	GOOD
13.	DUMB	-2	-1	0	+1	+2	SUPER
14.	GET IT	+2	+1	0	-1	-2	DON'T GET IT
15.	DISLIKE	-2	-1	0	+1	+2	LIKE
16.	WOW	+2	+1	0	-1	-2	YUK

APPENDIX C

LA MESA-SPRING VALLEY SCHOOL DISTRICT

MATH PARAPROFESSIONAL

Basic Policy. Under the supervision and guidance of the teacher, the paraprofessional shall be responsible for carrying out various assignments in the area of Mathematics, whether it be in or out of the classroom, for a specific period of time. An Instructional Math Aide need not perform such duties in the physical presence of the teacher, but the teacher shall retain his/her responsibility for the instruction and supervision of the pupil.

Area of Responsibility. The following suggestions have been prepared where a paraprofessional can be given the type of responsibility that would benefit the child's math needs as provided for by the teacher.

a. Clerical help.

1. Correcting math papers
2. Recording and filing math papers
3. Typing individual math worksheets

b. Preparing math materials.

1. Constructing supplementary math games
2. Running off math materials on the ditto machine
3. Preparing math drill sheets

c. Group instruction.

1. Working with two or more children in math, reviewing specific math concepts or introducing a different math concept to students, and using a variety of manipulative math objects for reinforcement.

d. Individual Instruction.

1. Working with an individual student in math, reviewing specific math concepts or introducing a different math concept to students, and using a variety of manipulative math objects for reinforcement.

e. Other duties.

1. Observing students during a math period
2. Planning and conferencing with the teacher
3. Testing students in math
4. Answering questions about math from students in the classroom
5. Set up math labs and bulletin boards

9/75
mw

APPENDIX D

LA MESA-SPRING VALLEY SCHOOL DISTRICT

ESEA TITLE III MATH PROJECT - PARAPROFESSIONAL

Teacher's Name	Grade Level	Name	Date

Please indicate the minutes (time) spent in each classroom for each day of the week. At the end of each week, please return this sheet to my office.

MON.	TUES.	WED.	THURS.	FRI.	CLERICAL HELP
					1. Correcting math papers.
					2. Recording and filing math papers.
					3. Typing individual math worksheets.
					4. *

					PREPARING MATH MATERIALS
					1. Constructing supplementary math games.
					2. Running off math materials on the ditto machine.
					3. Preparing math drill sheets.
					4. *

					GROUP INSTRUCTION
					1. Instructing the whole class.
					2. Working with two or more.
					3. Work with children using math materials.
					4. Using own math games.
					5. *

					INDIVIDUAL INSTRUCTION
					1. Reviewing specific math concepts.
					2. Introducing a math concept.
					3. Using manipulative math materials.
					4. *

					OTHER DUTIES
					1. Observing students during a math period.
					2. Planning and conferencing with the teacher.
					3. Testing students in math.
					4. Answering questions about math from students in class.
					5. Set up math labs and bulletin boards.
					6. *

*Other - please specify.

Please list each of the math materials you have used with students this week. (Ex., Base 10 blocks-small group - Monday; cuisenaire rods-individual help - Tuesday; basic text-circulating around classroom to help individual students - Friday.)

1/22/75

APPENDIX E

LA MESA-SPRING VALLEY SCHOOL DISTRICT

Teacher Evaluation Regarding the
EFFECTIVENESS of the PARAPROFESSIONAL on STUDENT ACHIEVEMENT in MATHEMATICS

School Year 1974 - 1975

Please circle type Aide: Trained Untrained

Grade Level: 4 - 5 - 6 - 7

1. In your math program how beneficial has the paraprofessional been in your classroom?

Very beneficial				Not Very beneficial
1	2	3	4	5

2. To what degree has the achievement of students in your class changed as the result of the paraprofessional?

Improved		Remained the same		Regressed
1	2	3	4	5

3. To what degree has the attitude of students in your class changed as the result of the paraprofessional?

Improved		Remained the same		Regressed
1	2	3	4	5

4. Rate the math abilities of the paraprofessional in your classroom.

Very Competent				Needs extensive training
1	2	3	4	5

5. Indicate the setting most prevalent in your classroom.

__________Total Group

__________Small Heterogeneous Group

__________Small Homogeneous Group

__________Totally Individualized

__________Other________________

____

6. Indicate the math program which is used in your classroom.

__________Dale Math

__________State Text

__________Sullivan

__________Other (specify________

____

______________________________)

TEACHER EVALUATION
Page Two

7. What has been your working relationship with your paraprofessional?

Excellent				Needs Improving
1	2	3	4	5

8. How beneficial has the paraprofessional's use of time been utilized in the classroom, as it relates to helping students?

Very Beneficial				Not very Beneficial
1	2	3	4	5

9. To what degree did the paraprofessional assist you in the preparation of math lesson plans?

Involved most of the time		Somewhat Involved		Not Involved
1	2	3	4	5

10. Could you implement the math program you are now using without the aid of a paraprofessional? YES NO

If not, why not?

11. Is it advantageous to have the paraprofessional working in a specific academic area? YES NO

Please specify advantages and disadvantages.

12. Does working with a paraprofessional allow you to use instructional strategies which would be difficult without assistance? YES NO

Please specify strategies.

13. Have there been any significant changes made in your classroom as a result of having a paraprofessional? YES NO

Please specify what changes, if any, were made.

14. Has there been an attitudinal change on your part from the time prior to having a Math Aide, and your present involvement with a Math Aide? Please specify what changes, if any, have taken place.

15. On the back side of this paper, make at least one recommendation to improve the Project for next year.

mw
5/9/75

APPENDIX F

LA MESA-SPRING VALLEY SCHOOL DISTRICT

Math Paraprofessional's Evaluation

School Year 1974 - 1975

Circle one: Trained Untrained

Grade Level 4 - 5 - 6 - 7

1. How beneficial do you think your involvement has been in the classroom?

Very Beneficial				Not very Beneficial	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

2. To what degree has the achievement of students changed as the result of your involvement in the classroom?

Improved		Remained the same		Regressed	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

3. To what degree has the attitude of students changed as a result of you being in the classroom?

Improved		Remained the same		Regressed	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

4. Rate the math competency of the teacher in each of the classrooms.

Very Competent				Not Competent	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

5. Indicate the setting most prevalent in each of the classrooms.

Teacher A	Teacher B	
____	____	Total Group
____	____	Small Heterogeneous Group
____	____	Small Homogeneous Group
____	____	Totally Individualized
____	____	(Other)____________________

6. Indicate the math program which is used in each of the classrooms.

Teacher A	Teacher B	
____	____	Dale Math
____	____	State Text
____	____	Sullivan
____	____	(Other - specify)______________________
		______________________________

7. What has been your working relationship with each teacher?

Excellent				Needs Improving	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

8. How beneficial has your time been utilized by the teacher?

Very Beneficial				Not very Beneficial	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

9. To what degree did the teacher spend with preparation of lesson plans?

Involved most of the time		Somewhat Involved		Not Involved	
1	2	3	4	5	Teacher A
1	2	3	4	5	Teacher B

10. Please make any additional comments that you feel were not covered.

APPENDIX G

MATH MATERIALS APPLICABLE FOR SEVERAL MATH WORKSHOPS

- Classroom Kit of Cuisenaire Rods
- Cubes, Squares, and Rods
- Geoboards
- Geoboard Activity Card
- Base Ten Blocks
- Arithmetic Skill Cards
- Developmental Math Cards
- Balance Scale
- Centimeter Cube
- Trundle Wheel
- Grid Paper Roll (One and two centimeter Rolls)
- Metric Plastic Stacking Weights
- Metric Graduated Beakers
- Vinyl Metric Rule
- Flexible Plastic Meter Rule
- Chip Trading Books: (I, II, III, IV, and V)
- Chips: Yellow (4), Blue (1), Green (2), Red (2) and Black (1)
- Desk Top Attribute Blocks
- Error Patterns In Computation (Paperback)
- Tangram
- Tangram Pieces
- Tangram Cards
- T.G. for Tangram
- Arithmetic Excursions (Paperback)
- Origins of Intellect (Paperback)
- Orbit Material for Math
- Pattern Blocks
- Operational Systems Game
- Math Mini-Lab
- Science and Math
- Angle Indicator
- Drill and Practice at the Problem Solving Level (Workbook)
- Nuffield Series Books (Paperback)
- Good Time Mathematics Series
- Elementary Balance and Weights
- Liquid Measure Sets
- Iron Weight
- Graduated Calipers
- Meter Rulers